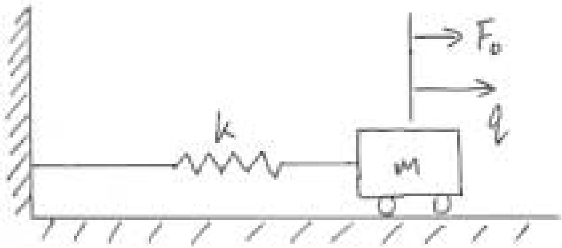
Solutions to Home Assignment #11

Practice Problems

1.

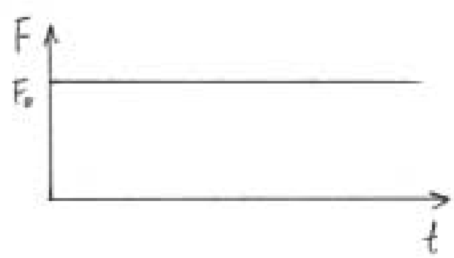


a) Response of system when subjected to rectangular pulse F_0 duration T_p.

The hint given in the problem suggests using the superposition principle to divide the pulse into a constant force F_0 applied at time 0, and a constant force $-F_0$ applied at time T_p.

The solution (or response) to each constant force applied individually to the system is as follows.

A. Constant force F_0 at time 0:



Governing equation:

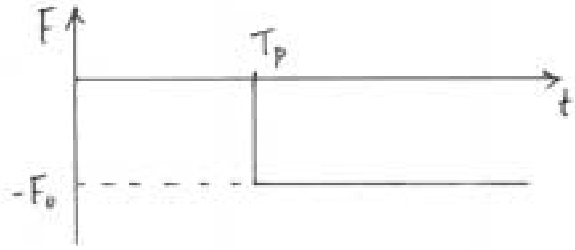
$$m\ddot{q} + kq = F_0$$

$$\Rightarrow \quad q(t) = \frac{F_0}{k}(1 - \cos\omega t) \quad \text{— ①}$$

$$\omega = \sqrt{\frac{k}{m}}$$

* unit #20, p.7

B. Constant force $-F_0$ at time T_p :



Governing equation

$$m\ddot{q} + kq = \begin{cases} 0 & t < T_p \\ -F_0 & t \geq T_p \end{cases}$$

The solution to the governing equation for $t \geq T_p$ can be found by shifting the time frame in the solution for case A. Note that for $t < T_p$, the system is not excited, and therefore, the response is zero.

$$\Rightarrow \quad q = \begin{cases} 0 & t < T_p \\ -\frac{F_0}{k}\left(1 - \cos\omega(t - T_p)\right) & t \geq T_p \end{cases} \quad \text{—②}$$

Superposing equations ① and ②, we get

$$q(t) = \begin{cases} \frac{F_0}{k}(1 - \cos\omega t) & t < T_p \\ \frac{F_0}{k}\left[-\cos\omega t + \cos\omega(t - T_p)\right] & t \geq T_p \end{cases} \quad \text{—③}$$

$$\| $$

$$\frac{F_0}{k} 2 \sin\frac{\omega}{2}(2t - T_p) \sin\frac{1}{2}\omega T_p$$

* Note : $\cos v - \cos u = 2\sin\frac{u+v}{2}\sin\frac{u-v}{2}$

b) From equation ③ we can see that the responce for $t < T_p$ is simply the responce for a constant force F_0 applied at time 0.

The response for $t \geq T_p$ is more interesting. Re-writing the response for $t \geq T_p$,

$$q(t) = \underbrace{\frac{2F_0}{k}\sin\frac{1}{2}\omega T_p}_{\text{coefficient independent of time}} \overbrace{\sin\frac{\omega}{2}(2t - T_p)}^{\text{time dependent}} \quad (t \geq T_p) \qquad \text{—— ④}$$

From equation ④, we can immediately see some interesting responses for particular T_p's due to the coefficient independent of time.

A. $T_p = 0$: $\quad \frac{2F_0}{k}\sin\frac{1}{2}\omega T_p = 0$

coefficient

$\therefore \; q(t) = 0 \quad (t \geq T_p)$ — This is expected because if $T_p = 0$, no force is being applied.

B. $T_p = \frac{2\pi}{\omega} n$ (integer) : $\quad \frac{2F_0}{k}\sin\frac{1}{\cancel{2}}\cancel{\omega}\left(\frac{\cancel{2}\pi}{\cancel{\omega}}n\right) = \frac{2F_0}{k}\sin \pi n = 0$

coefficient

$\therefore \; q(t) = 0 \quad (t \geq T_p)$

C. $T_p = \frac{\pi}{\omega} n$ (odd integer) : $\quad \frac{2F_0}{k}\sin\frac{1}{2}\cancel{\omega}\left(\frac{\pi}{\cancel{\omega}}n\right) = \frac{2F_0}{k}\sin\frac{1}{2}\pi n$

$$\therefore \; q(t) = \frac{2F_0}{k}\sin\frac{\omega}{2}(2t - T_p)\sin\frac{1}{2}\pi n \quad (t \geq T_p)$$

$$\Rightarrow q(t) = \begin{cases} \frac{2F_0}{k}\sin\frac{\omega}{2}(2t - T_p) & n = 1, 5, 9 \ldots \\ -\frac{2F_0}{k}\sin\frac{\omega}{2}(2t - T_p) & n = 3, 7, 11 \ldots \end{cases}$$

To plot the response, let's first normalize with respect to $\frac{2\pi}{\omega}$, which is the period. Thus, from equation 3 (& 4),

$$q(t) = \begin{cases} \frac{F_0}{k}\left(1-\cos 2\pi \frac{T_p}{\left(\frac{2\pi}{\omega}\right)}\left(\frac{t}{T_p}\right)\right) & \left(\frac{t}{T_p}<1\right) \\ \frac{2F_0}{k}\sin\pi\frac{T_p}{\left(\frac{2\pi}{\omega}\right)}\sin\pi\frac{T_p}{\left(\frac{2\pi}{\omega}\right)}\left(\frac{2t}{T_p}-1\right) & \left(\frac{t}{T_p}\geq 1\right) \end{cases} \quad \text{— ⑤}$$

If we define

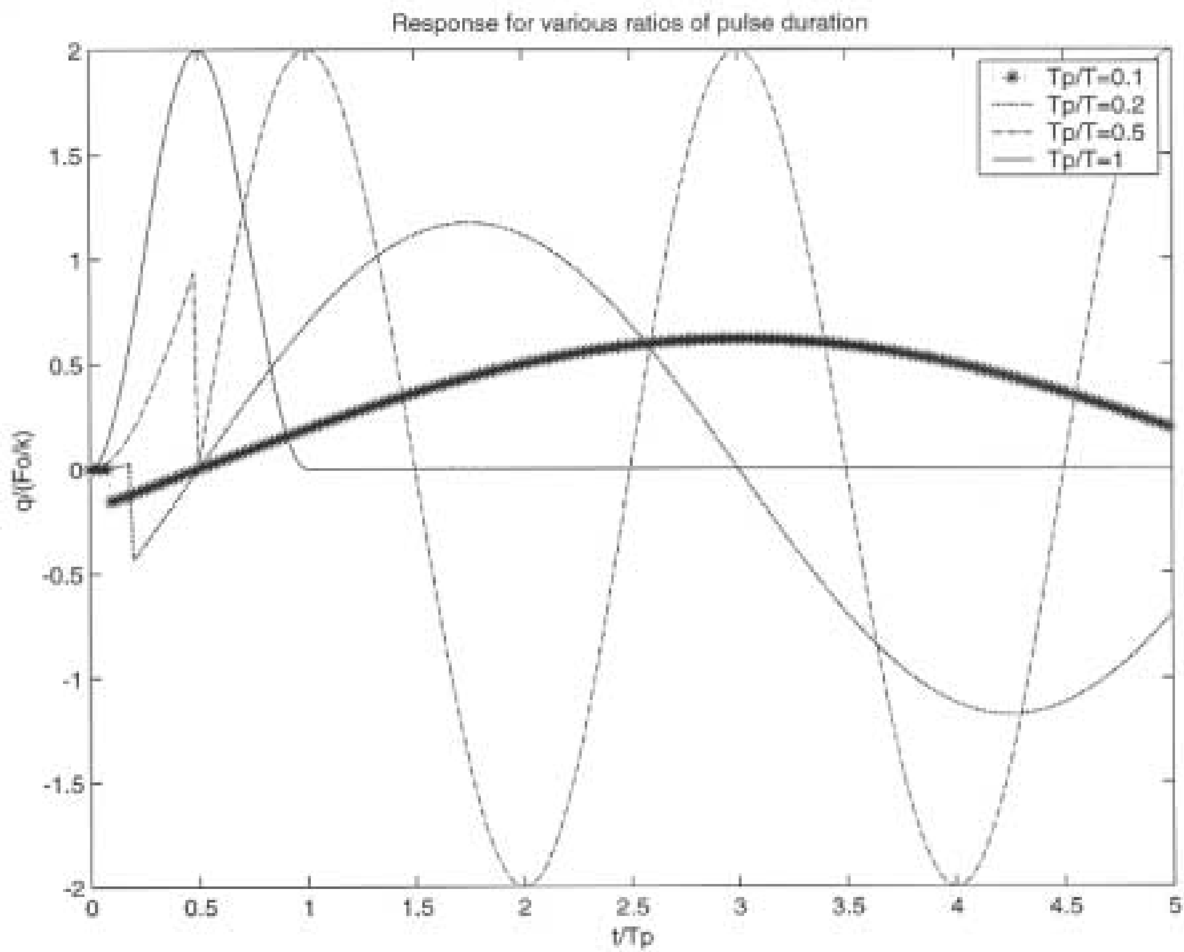
$$\frac{T_p}{\left(\frac{2\pi}{\omega}\right)} = \tau \quad , \quad \frac{t}{T_p} = t'$$

and normalized the response by F_0/k, we get

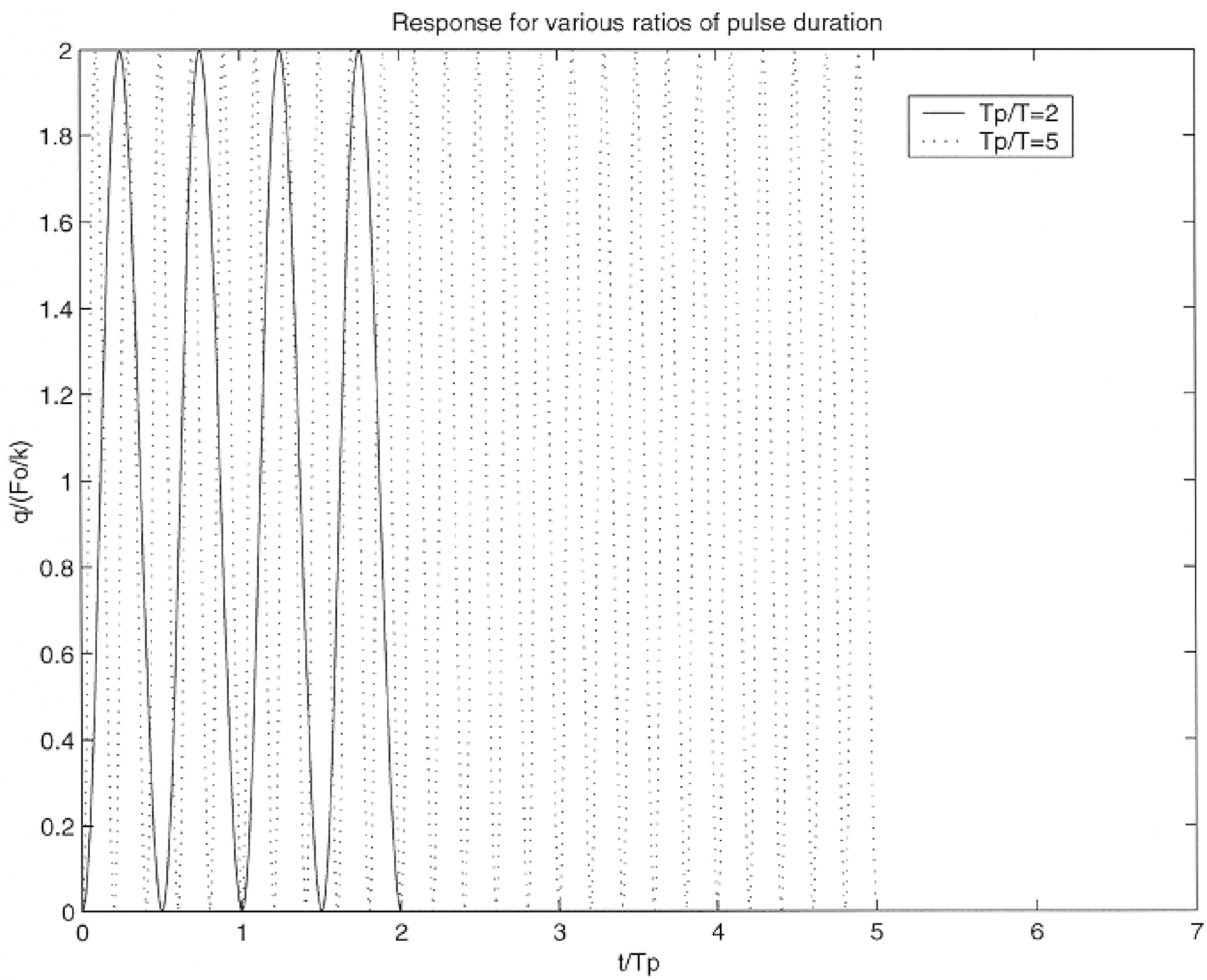
$$\frac{q(t)}{(F_0/k)} = \begin{cases} (1-\cos 2\pi\tau t') & t'<1 \\ 2\sin\pi\tau\sin\pi\tau(2t'-1) & t'\geq 1 \end{cases} \quad \text{— ⑥}$$

The plots for $T_p/\left(\frac{2\pi}{\omega}\right) = 0.1, 0.2, 0.5$ & 1 are shown on p. 5, and for $T_p/\left(\frac{2\pi}{\omega}\right) = 2$ and 5 are shown on p. 6. For values of the ratio of pulse duration to natural frequency $\left(= T_p/\left(\frac{2\pi}{\omega}\right)\right)$ not equal to an integer value, the mass oscillates with a frequency of ω until $t = T_p$, after which it oscillates with the same frequency but different amplitude ranges. The amplitude before $t = T_p$ is between 0 and $2F_0/k$, while after $t = T_p$, the amplitude is $2\sin\pi\tau\left(\frac{F_0}{k}\right)$.

For integer values of $T_p/(\frac{2\pi}{\omega})$, the response before $t=T_p$ is the same as before (i.e., oscillates with frequency ω between 0 and $2F_0/k$), but after $t=T_p$, the response is zero. This also follows from

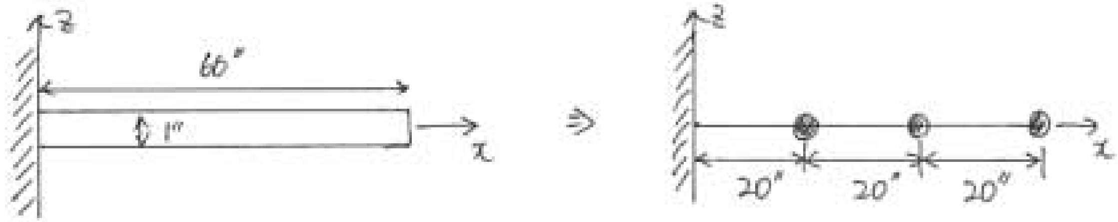


Case B where, for $T_p = \frac{2\pi}{\omega} n$ with any integer, n, $q(t)=0$ for $t \geq T_p$. The case for $T_p/(\frac{2\pi}{\omega}) = 10$ is not shown, but is very similar to the other integer cases.



Response for various ratios of pulse duration
Tp/T=2
Tp/T=5
q/(Fo/k)
t/Tp
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
0
1
2
3
4
5
6
7

2.



$E = 10.0$ Msi

$\rho = 0.1$ lb/in^3

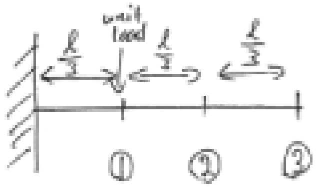
$I = \frac{1}{12} 1^4 = 0.083$ in^4

Represent the continous system as a three-mass system and determine the natural frequencies and associated mode shapes. So, the first thing we have to do is to find a way to discretize the continuous beam into the three-mass system. We discussed how we could do this in unit #22 (from p. 10 ~). The fundamental idea is to obtain the flexibility influence coefficients at each discretized point and from that, invert to obtain the stiffness influence coefficients. The flexibility influence coefficients can be obtained from

$$c_{ij} = \frac{1}{2EI}\left(x_i^2 x_j - \frac{x_i^3}{3}\right) \qquad \text{(unit\#21, p10)} \quad (1)$$

Once we obtain the stiffness influence coefficients, we need to discretize the mass of the beam into three concentrated masses. This will allow us to obtain a set of second-order differential equations, which are the governing equations for the three-mass system. The natural frequencies and associated mode shapes can be obtained from that equation.

a) Determine flexibility influence coefficients :



- for unit load applied at station 1 :

$$C_{11} = \frac{1}{2EI}\left(\frac{\ell^3}{27} - \frac{1}{3}\frac{\ell^3}{27}\right) = \frac{1}{\cancel{2}EI}\frac{\cancel{2}\ell^3}{3\cdot 27}$$

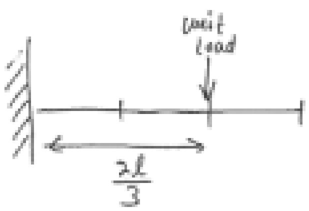
displacement at 1 due to unit load at 1

$$= \frac{1}{3\cdot 27EI}\ell^3 \qquad \text{—— (2)}$$

displacement at 2 due to unit load at 1

$$C_{21} = C_{12} = \frac{1}{2EI}\left(\left(\frac{\ell^2}{9}\right)\frac{2\ell}{3} - \frac{1}{3}\frac{\ell^3}{27}\right) = \frac{1}{2EI}\frac{5\ell^3}{3\cdot 27} = \frac{5}{162EI}\ell^3 \qquad \text{—— (3)}$$

$$C_{31} = C_{13} = \frac{1}{2EI}\left(\left(\frac{\ell^2}{9}\right)\ell - \frac{1}{3}\frac{\ell^3}{27}\right) = \frac{1}{\cancel{2}EI}\frac{\overset{4}{\cancel{8}}\ell^3}{3\cdot 27} = \frac{4}{3\cdot 27EI}\ell^3 \qquad \text{—— (4)}$$

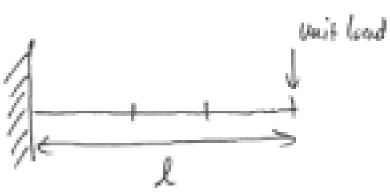


- for unit load applied at station 2 :

$$C_{22} = \frac{1}{2EI}\left(\frac{8\ell^3}{27} - \frac{1}{3}\frac{8\ell^3}{27}\right) = \frac{1}{\cancel{2}EI}\frac{\cancel{2}8\ell^3}{3\cdot 27}$$

$$= \frac{8}{3\cdot 27EI}\ell^3 \qquad \text{—— (5)}$$

$$C_{32} = C_{23} = \frac{1}{2EI}\left(\left(\frac{4\ell^2}{9}\right)\ell - \frac{1}{3}\frac{8\ell^3}{27}\right) = \frac{1}{\cancel{2}EI}\frac{\overset{14}{\cancel{28}}\ell^3}{3\cdot 27} = \frac{14}{3\cdot 27EI}\ell^3 \qquad \text{—— (6)}$$



- for unit load applied at station 3

$$C_{33} = \frac{1}{2EI}\left(\ell^3 - \frac{1}{3}\ell^3\right) = \frac{1}{3EI}\ell^3 \qquad \text{—— (7)}$$

Using symmetry, (i.e. $C_{ij} = C_{ji}$), we can write the flexibility influence coefficients in matrix form as,

$$\underset{\sim}{C} = \frac{\ell^3}{EI}\begin{bmatrix} \frac{1}{81} & \frac{5}{162} & \frac{4}{81} \\ & \frac{8}{81} & \frac{14}{81} \\ \text{SYM} & & \frac{1}{3} \end{bmatrix} \quad \text{— ⑧}$$

Plugging in the given values for $\ell = 60''$, $E = 10.0$ Msi, $I = 0.083 in^4$,

$$\underset{\sim}{C} = \begin{bmatrix} 3.20\times10^{-3} & 8.00\times10^{-3} & 1.30\times10^{-2} \\ & 2.60\times10^{-2} & 4.50\times10^{-2} \\ \text{SYM} & & 8.10\times10^{-2} \end{bmatrix} \quad \text{— ⑨}$$

*units of $\frac{in}{lb}$

Inverting the matrix in equation ⑨, we obtain the stiffness influence coefficient matrix (using Matlab)

$$\underset{\sim}{k} = \begin{bmatrix} 1.40\times10^{+3} & -1.10\times10^{+3} & 2.9\times10^{2} \\ & 1.10\times10^{3} & -3.80\times10^{2} \\ \text{SYM} & & 1.70\times10^{2} \end{bmatrix} \quad \text{— ⑩}$$

*units of $\frac{lb}{in}$

b) Mass discretization: The total mass of the beam is

$$M = \rho V = (0.1\ lbs/in^3)(60''\times1''\times1'')/(32.2\ ft/s^2 \cdot 12\ in/ft)$$

$$\Rightarrow \quad M = 0.016 \text{ slugs}$$

Since we discretized the beam as three masses spaced $\frac{\ell}{3}$ from each other, the equivalent mass, m_i, of each mass should be:

$$m = m_1 = m_2 = m_3 = \frac{M}{3} = 0.0053 \text{ slugs}.$$

In matrix form,

$$\underset{\sim}{m} = \begin{bmatrix} 0.0053 & 0 & 0 \\ & 0.0053 & 0 \\ \text{SYM} & & 0.0053 \end{bmatrix} \text{slugs} \qquad \text{—— } (11)$$

c) Governing equation and the natural frequencies and modes:

equivalent stiffness (= stiffness influence coefficient matrix

$$\underset{\sim}{m}\ddot{\underset{\sim}{q}} + \underset{\sim}{k}\underset{\sim}{q} = \underset{\sim}{0} \qquad \text{—— } (13)$$

mass matrix $\quad q$ is deflection (thus $\ddot{q}$ is acceleration)

The mass matrix is given in equation (11), the stiffness matrix in equation (10) and the deflection vector is

$$\underset{\sim}{q} = \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \end{Bmatrix}$$

The natural frequencies can be found by solving equation (13) for the eigenvalues.

$$|\underset{\sim}{k} - \omega^2 \underset{\sim}{m}| = \begin{vmatrix} 1900 - \omega^2(0.0053) & -1100 & 290 \\ & 1100 - \omega^2(0.0053) & -380 \\ \text{SYM} & & 170 - \omega^2(0.0053) \end{vmatrix} = 0$$

Solving using Matlab, we get

$$\boxed{\begin{aligned} \omega_1 &= 40.9 \text{ rad/s} \\ \omega_2 &= 270 \text{ rad/s} \\ \omega_3 &= 120 \text{ rad/s} \end{aligned}}$$

* Check units of k and $\omega^2 m$: $[k] = \left[\frac{lb}{in}\right] = [\text{slugs}]\left[\frac{in}{s^2}\right]\left[\frac{1}{in}\right] = \left[\frac{\text{slugs}}{s^2}\right]$

$[\omega^2 m] = \left[\frac{\text{slugs}}{s^2}\right]$

* Note: When we solve for the frequency, ω_r, we get the positive and negative values of the same number, e.g., $\omega_1 = \pm 40.9$ rad/s. However, negative frequency is not physically possible, so it is discarded.

To find the eigenvector associated with each eigenvalue, we substitute in ω_r and solve for the vector $\underset{\sim}{A}$.

$$[\underset{\sim}{k} - \omega_r^2 \underset{\sim}{m}]\underset{\sim}{A} = 0 \quad \text{(unit \#22 p4)}$$

For $\omega_r = \omega_1 = 40.9$ rad/s ,

$$[\underset{\sim}{k} - \omega_1^2 \underset{\sim}{m}]\,\underset{\sim}{A} = \begin{bmatrix} 1900 & -1100 & 290 \\ & 1050 & -380 \\ \text{SYM} & & 160 \end{bmatrix} \begin{Bmatrix} A_1/A_3 \\ A_2/A_3 \\ 1 \end{Bmatrix} = \underset{\sim}{0}$$

$$\Rightarrow \quad \begin{bmatrix} 1900 & -1100 \\ -1100 & 1050 \end{bmatrix} \begin{Bmatrix} A_1/A_3 \\ A_2/A_3 \end{Bmatrix} = \begin{Bmatrix} -290 \\ 380 \end{Bmatrix}$$

$$\Rightarrow \quad A_1/A_3 = 0.150$$

$$A_2/A_3 = 0.51$$

For $\omega_r = \omega_2 = 270$ rad/s

$$[\underset{\sim}{k} - \omega_2^2 \underset{\sim}{m}]\,\underset{\sim}{A} = \begin{bmatrix} 1500 & -1100 & 290 \\ & 610 & -380 \\ \text{SYM} & & -220 \end{bmatrix} \begin{Bmatrix} A_1/A_3 \\ A_2/A_3 \\ 1 \end{Bmatrix} = \underset{\sim}{0}$$

$$\Rightarrow \quad \begin{bmatrix} 1500 & -1100 \\ -1100 & 610 \end{bmatrix} \begin{Bmatrix} A_1/A_3 \\ A_2/A_3 \end{Bmatrix} = \begin{Bmatrix} -290 \\ 380 \end{Bmatrix}$$

$$\Rightarrow \quad A_1/A_3 = -1.1$$

$$A_2/A_3 = -1.2$$

For $\omega_r = \omega_3 = 120$ rad/s

$$[k - \omega_3^2 m] = \begin{bmatrix} -830 & -1100 & 290 \\ & -1700 & -380 \\ \text{SYM} & & -2600 \end{bmatrix} \begin{Bmatrix} A_1/A_3 \\ A_2/A_3 \\ 1 \end{Bmatrix} = \underset{\sim}{0}$$

$$\Rightarrow \begin{bmatrix} -830 & -1100 \\ -1100 & -1700 \end{bmatrix} \begin{Bmatrix} A_1/A_3 \\ A_2/A_3 \end{Bmatrix} = \begin{Bmatrix} -290 \\ 380 \end{Bmatrix}$$

$$\Rightarrow \quad A_1/A_3 = 4.5$$

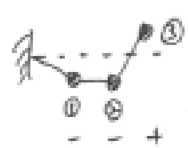
$$A_2/A_3 = -3.2$$

Thus, the eigenvalues and their associated mode shapes can be summarized as follows.

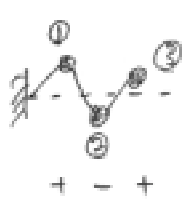
$$\boxed{\begin{array}{ll} \omega_1 = 40.9 \text{ rad/s} & \underset{\sim}{\phi}_1 = \begin{Bmatrix} 0.150 \\ 0.51 \\ 1 \end{Bmatrix} \\ \omega_2 = 210 \text{ rad/s} & \underset{\sim}{\phi}_2 = \begin{Bmatrix} -1.1 \\ -1.2 \\ 1 \end{Bmatrix} \\ \omega_3 = 120 \text{ rad/s} & \underset{\sim}{\phi}_3 = \begin{Bmatrix} 4.5 \\ -3.2 \\ 1 \end{Bmatrix} \end{array}}$$

$\Rightarrow$

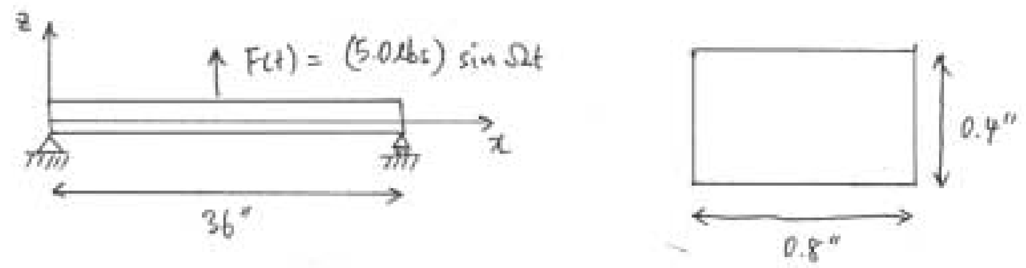
$\Rightarrow$



$\Rightarrow$



3.



$E = 10.0\,Msi$

$\rho = 0.1\,lb/in^3$

Sinusoidal force applied at center of simply-supported beam.

a) For a continuous beam, the governing equation is

$$EI\frac{d^4 w}{dx^4} + m\ddot{w} = p_z \quad \text{——— ①}$$

\# unit #33, p3

The general solution for this differential equation is

unit #33, p6

$$w(x,t) = \bar{w}(x)e^{i\omega t}$$

$$= [C_1 \sinh\lambda x + C_2\cosh\lambda x + C_3\sin\lambda x + C_4\cos\lambda x]e^{i\omega t} \quad \text{— ②}$$

unit #33, p7

This equation needs 4 boundary conditions for $C_1 \sim C_4$, which are

\# The following derivation is also discussed in unit #13

$$@\ x = 0,\ l \quad \begin{cases} w = 0 \\ M = EI\dfrac{d^2 w}{dx^2} = 0 \end{cases} \quad \text{——— ③}$$

Plugging equation ② into equation ③, we get four equations. If we express these equations in matrix form, we obtain

$$\begin{array}{l} w(0)=0 \rightarrow \\ w''(0)=0 \rightarrow \\ w(l)=0 \rightarrow \\ w''(l)=0 \rightarrow \end{array} \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ \sinh\lambda l & \cosh\lambda l & \sin\lambda l & -\cos\lambda l \\ \sinh\lambda l & \cosh\lambda l & -\sin\lambda l & -\cos\lambda l \end{bmatrix} \begin{Bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{Bmatrix} = \underset{\sim}{0} \quad \text{——— ④}$$

The determinant of the matrix in equation 4 is $C_3 \sin \lambda l$, and setting this to zero, to get non-trivial solutions, we get

$$C_3 \sin \lambda l = 0$$

$$\Rightarrow \quad \lambda l = n\pi$$

$$\Rightarrow \quad \frac{m\omega^2}{EI} = \frac{n^4\pi^4}{l}$$

$$\Rightarrow \quad \omega = n^2\pi^2\sqrt{\frac{EI}{ml^4}}$$

$$\therefore \; \omega_r = r^2\pi^2\sqrt{\frac{EI}{ml^4}} \qquad \text{—— } 5$$

As for multi-mass systems, the associated modes (or eigenvectors), are found by plugging the frequency, ω_r, back into the matrix governing equation. We find that

$$\phi_r = \sin\frac{r\pi x}{l} \qquad \text{for } r = 1, 2, 3, \cdots \qquad \text{—— } 6$$

From equations 5 and 6, the first three frequencies and modes are

$$\omega_1 = \pi^2\sqrt{\frac{EI}{ml^4}} \qquad \phi_1 = \sin\frac{\pi x}{l}$$

$$\omega_2 = 4\pi^2\sqrt{\frac{EI}{ml^4}} \qquad \phi_2 = \sin\frac{2\pi x}{l}$$

$$\omega_3 = 9\pi^2\sqrt{\frac{EI}{ml^4}} \qquad \phi_3 = \sin\frac{3\pi x}{l}$$

The given data for the beam is: $E = 10.0\,\text{Msi}$, $l = 36''$,

$$I = \frac{1}{12}(0.8'')(0.4'')^3 = 4.27\times10^{-3}\ \text{in}^4$$

$$m = \frac{\rho}{g}\cdot(\text{volume})/(\text{length})$$

$$= \frac{0.1\ \text{lb/in}^3}{32.2\ \text{ft/s}^2\cdot 12\ \text{in/ft}}(0.8'')(0.4'')(36'')/(36'')$$

$$= 8.29\times10^{-5}\ \text{slugs/in}$$

* Note: $m = \left[\frac{\text{mass}}{\text{length}}\right]$ according to definition in unit #23 p2.

$\therefore \sqrt{\frac{EI}{ml^4}}$ has units of $\sqrt{\frac{(\text{psi})(\text{in}^4)}{\text{slugs/in}\cdot\text{in}^4}} = \frac{1}{s}$

Plugging these values into the frequencies and mode shapes, we get

$$\boxed{\begin{array}{ll} \omega_1 = 173\ \text{rad/s} & \phi_1 = \sin\frac{\pi x}{36} \\ \omega_2 = 691\ \text{rad/s} & \phi_2 = \sin\frac{\pi x}{18} \\ \omega_3 = 1560\ \text{rad/s} & \phi_3 = \sin\frac{\pi x}{12} \end{array}}$$

b) The normal equations of motion have the form

* unit #23 p14

$$M_r\ddot{\xi}_r + M_r\omega_r^2\xi_r = \Xi_r$$

where

$$M_r = \int_0^l m\phi_r^2\,dx \quad , \quad \Xi_r = \int_0^l \phi_r p_z(x,t)\,dx$$

generalized mass of rth mode

generalized force of rth mode

The applied load for this problem is

$$p_z(x,t) = F(t)\,\delta(x - 18'')$$

Therefore, the generalized force becomes

$$\Xi_r = \int_0^l \phi_r F(t)\,\delta(x - 18'')\,dx$$

$$= \phi_r(18'')F(t) \qquad \text{---------} \quad ①$$

The generalized mass matrix can be evaluated as follows.

$$* \int \sin^2 x\,dx = \tfrac{1}{2}x - \tfrac{1}{4}\sin 2x + c$$

$$M_1 = (8.29\times10^{-5}\text{ slugs/in})\int_0^{l=36''} \sin^2\frac{\pi x}{36}\,dx$$

$$= (8.29\times10^{-5}\text{ slugs/in})\left[\frac{1}{2}\frac{\pi x}{36} - \frac{1}{4}\sin\frac{\pi x}{18}\right]_0^{36}$$

$$= 1.30\times10^{-4}\text{ slugs}$$

$$M_2 = (8.29\times10^{-5}\text{ slugs/in})\int_0^{l=36''} \sin^2\frac{\pi x}{18}\,dx$$

$$= (8.29\times10^{-5}\text{ slugs/in})\left[\frac{1}{2}\frac{\pi x}{18} - \frac{1}{4}\sin\frac{\pi x}{9}\right]_0^{36}$$

$$= 2.60\times10^{-4}\text{ slugs}$$

$$M_3 = (8.29\times10^{-5}\text{ slugs/in})\int_0^{l} \sin^2\frac{\pi x}{12}\,dx$$

$$= (8.29\times10^{-5}\text{ slugs/in})\left[\frac{1}{2}\frac{\pi x}{12} - \frac{1}{4}\sin\frac{\pi x}{9}\right]_0^{36}$$

$$= 3.91\times10^{-4}\text{ slugs}$$

The generalized force is

$$\Xi_1 = \sin\frac{\pi(18")}{36"} F(t) = F(t)$$

$$\Xi_2 = \sin\frac{\pi(18")}{18"} F(t) = 0$$

$$\Xi_3 = \sin\frac{\pi(18")}{12"} F(t) = -F(t)$$

Thus, the normal equations of motion for the first three modes are:

$$\boxed{\begin{aligned} 0.000130\,\ddot{\xi}_1 + 3.88\,\xi_1 &= F(t) \\ 0.000260\,\ddot{\xi}_2 + 124\,\xi_2 &= 0 \\ 0.000391\,\ddot{\xi}_3 + 947\,\xi_3 &= -F(t) \end{aligned}}$$

c) From b), we know that the general form of the governing decoupled equation is

$$M_r\ddot{\xi}_r + M_r\omega_r^2\xi_r = \phi_r(18")(5.0\,\text{lbs})\sin\Omega t$$

The particular solution to this differential equation was obtained in unit #20. p22.

$$\xi_r = \frac{\phi_r(18")(5.0\,\text{lbs})}{M_r\omega_r^2\left[1-\left(\frac{\Omega}{\omega_r}\right)^2\right]}\sin\Omega t \qquad \text{(E)}$$

For $r=1, 2, 3$, ξ_r is

$$\xi_1 = \frac{1\,(5.0\,lbs)}{(3.88\,\frac{slug}{s^2})[1-(\frac{\Omega}{\omega_1})^2]}\sin\Omega t = \frac{1.29}{[1-(\frac{\Omega}{173})^2]}\sin\Omega t \quad (in) \qquad \text{—— ⑨}$$

$$\xi_2 = \frac{0\,(5.0\,lbs)}{(124\,\frac{slug}{s^2})[1-(\frac{\Omega}{\omega_2})^2]} = 0 \qquad \text{—— ⑩}$$

$$\xi_3 = \frac{-1\,(5.0\,lbs)}{(947\,\frac{slug}{s^2})[1-(\frac{\Omega}{\omega_3})^2]}\sin\Omega t = \frac{-0.00528}{[1-(\frac{\Omega}{1560})^2]}\sin\Omega t \quad (in) \qquad \text{—— ⑪}$$

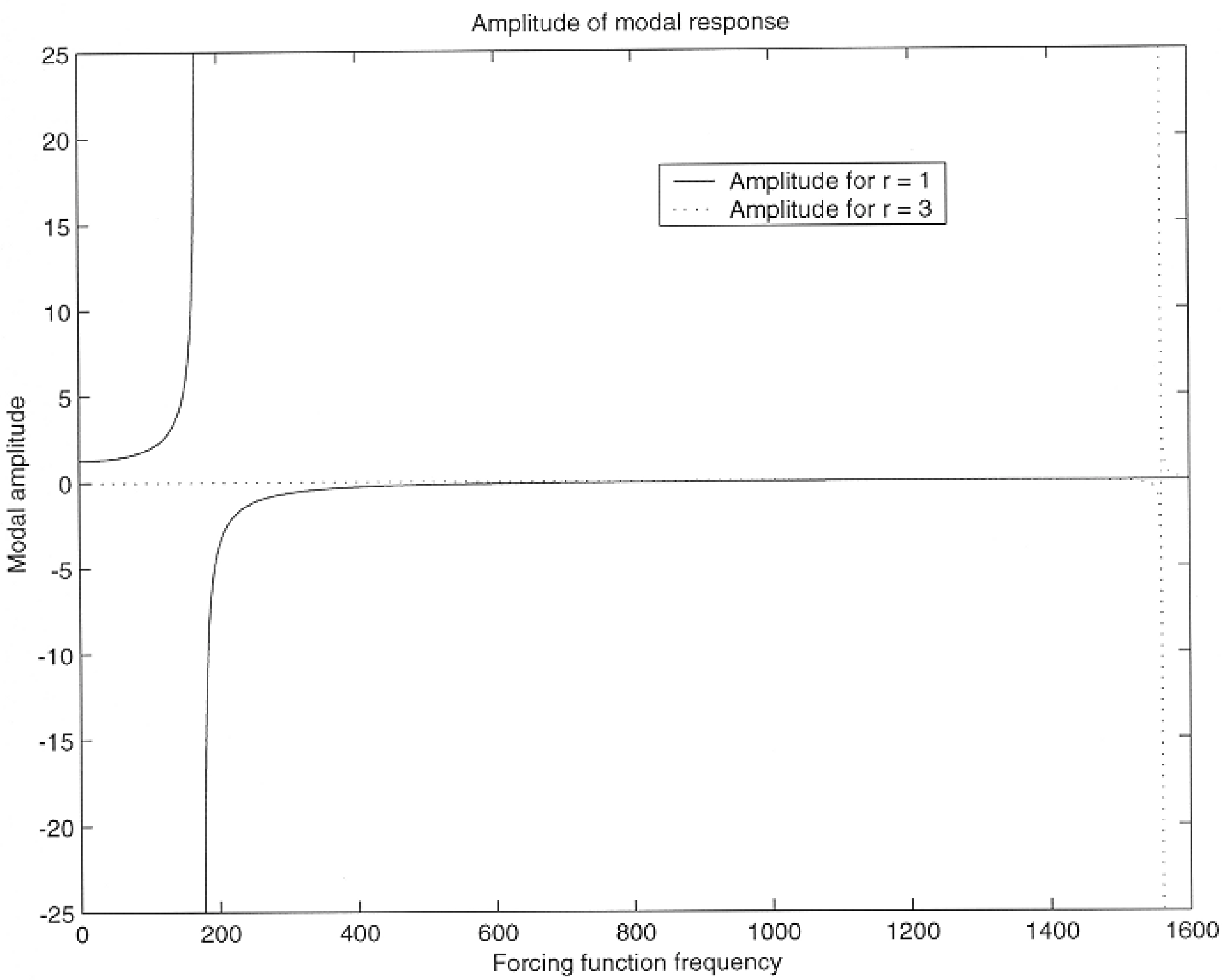
The deflection at the center is

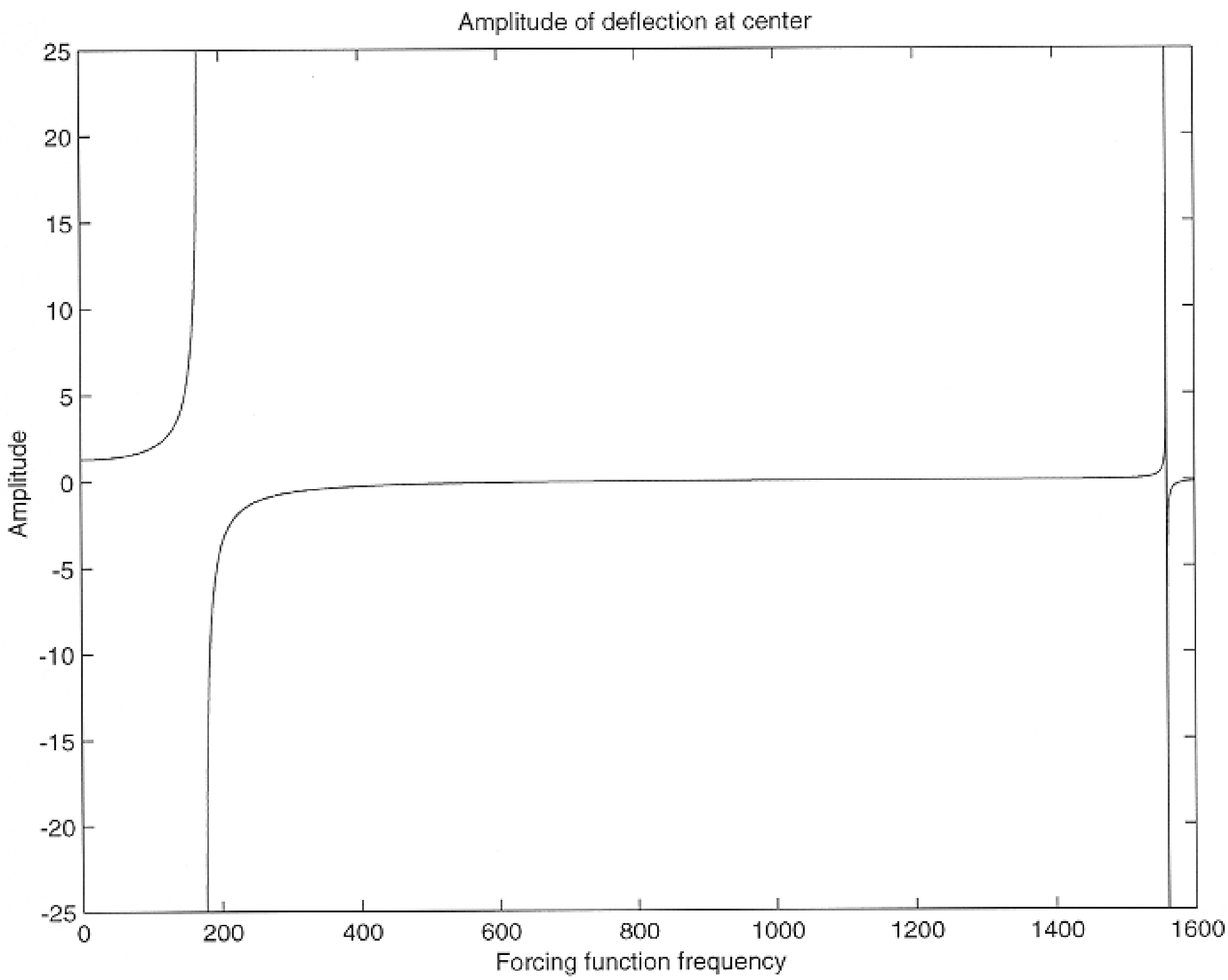
$$w(\tfrac{l}{2}, t) = \sum_{r=1}^{3}\phi_r(\tfrac{l}{2})\,\xi_r(t) = \phi_1(\tfrac{l}{2})\xi_1(t) + \phi_2(\tfrac{l}{2})\overset{=0}{\xi_2(t)} + \phi_3(\tfrac{l}{2})\xi_3(t)$$

$$= \sin\Omega t\left[\frac{1.29}{[1-(\frac{\Omega}{173})^2]} + \frac{0.00528}{[1-(\frac{\Omega}{1560})^2]}\right]$$

$$\Rightarrow \boxed{w(18'', t) = \sin\Omega t\left[\frac{1.29}{[1-(\frac{\Omega}{173})^2]} + \frac{0.00528}{[1-(\frac{\Omega}{1560})^2]}\right]} \qquad \text{—— ⑫}$$

d) The amplitudes of the modal responses are plotted on the next page. The amplitudes become very large and goes to infinity when the natural frequencies ω_1 and ω_3 are approached.


Amplitude of modal response
Amplitude for r = 1
Amplitude for r = 3
Modal amplitude
Forcing function frequency
25
20
15
10
5
0
-5
-10
-15
-20
-25
0
200
400
600
800
1000
1200
1400
1600


Amplitude of deflection at center
25
20
15
10
5
0
-5
-10
-15
-20
-25
Amplitude
0
200
400
600
800
1000
1200
1400
1600
Forcing function frequency