Lecture 28

Nov 17, 03.

7.3> Integral Methods for Turb. flows

A> Two Eqn. method closure relations

B> Lag Effects.

<u>Recall</u>

G-β locus for equilibrium flows.

$$\frac{G^2}{A^2} = 1 + B\beta$$

which gave us quantitative relationship between C_f, $\frac{dp}{dx}$, H

<u>Recall</u> IBL eqns:

$$\frac{d\theta}{dx} = C_f/2 - (H+2)\frac{\theta}{u_e}\frac{du_e}{dx}$$

$$\frac{\theta}{H^*}\frac{dH^*}{dx} = \frac{2C_D}{H^*} - C_f/2 + (H-1)\frac{\theta}{u_e}\frac{du_e}{dx}$$

$$C_D \equiv \frac{1}{\rho u_e^3}\int_{y=0}^{\delta} \tau\frac{\partial u}{\partial y}\,dy = \frac{1}{\rho u_e^3}\int_{y=0}^{\delta}(\mu+\mu_t)\left(\frac{\partial u}{\partial y}\right)^2 dy$$

$$\int\left[\mu\left(\frac{\partial u}{\partial y}\right)^2 + (-\rho\overline{u'v'})\frac{\partial u}{\partial y}\right]dy.$$

We require C_f, C_D, H^* closure relationships.

(1) Turbulent C_f: We are given

$$\frac{u_e - u}{u^*} = f(y/\delta^*, \beta)$$

$$\frac{u}{u^*} = g(y^+, k^+)$$

$$u^+ - \sqrt{\frac{2}{C_f}} = f(y/\delta, \beta) \quad \text{— Coles}$$

$$u^+ = g(\delta^+) + C(\beta)$$

$$= g\left(Re_\delta \cdot \sqrt{C_f/2}\right) + C(\beta)$$

$\downarrow H, C_f$

$$\frac{U_e}{u^*} = \sqrt{\frac{2}{C_f}} = \frac{1}{K}\ln \delta^+ + B + C(\beta)$$

$\therefore \quad C_f = C_f(Re_\theta, H)$ — functional form. Obtain expression from curve fit from different values of H, Re_θ. See handout eqn (6.17)

(2) H^*

Laminar $\quad H^* = H^*(H)$

Turbulent $\quad H^* = H^*(H; Re_\theta)$

weak

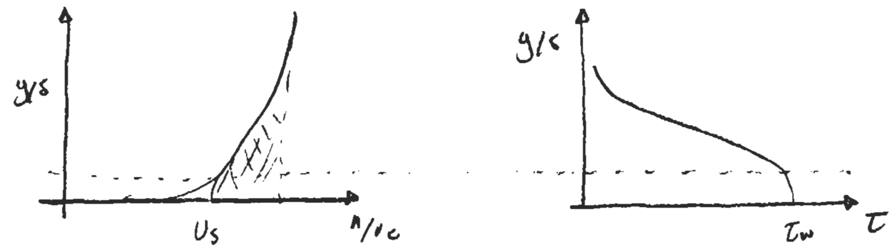
(3) C_D

$$C_f = C_f(H, Re_\theta) \quad \text{and} \quad H^* = H^*(H, Re_\theta)$$

$$\Rightarrow \quad C_D = C_D(H, Re_\theta) = ?$$

$$C_D = \frac{1}{\rho U_e^3}\int_0^\infty \tau \frac{\partial u}{\partial y}\, dy$$

can be separated into inner + outer contribution



$$C_{D_i} = \frac{1}{\rho u_e^3}\int_0^{u_s/u_e} \tau \, du \approx \frac{C_f}{2}\cdot u_s$$

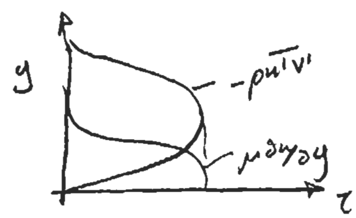
$$C_{D_o} = \frac{1}{\rho u_e^3}\int_0^{\infty} \tau \frac{\partial u}{\partial y}\, dy$$

outer eddy viscosity model

Coles profile

$$C_D \equiv \frac{1}{\rho U_e^3}\int_0 \left(\mu\frac{\partial u}{\partial y} - \rho\overline{u'v'}\right)\frac{\partial u}{\partial y}\, dy$$

suggest splitting C_D into inner and outer contributions



$$C_{D_o} \approx \frac{1}{\rho u_e^3}\cdot\frac{\pi}{4}\,\tau_{max}\,(u_e - u_s)$$

Combining these we get

$$C_D = \underbrace{\frac{C_f}{2}U_s}_{\text{inner}} + \underbrace{\frac{\pi}{4}C_\tau(1-U_s)}_{\text{outer}}$$

We can also arrive at this result using concept of equilibrium flow

G = const , $\frac{dG}{dx}$ = small for turbulent flow (slow change)

$\therefore \quad \frac{dG}{dx} = 0 \quad \Rightarrow \quad \frac{dH}{dx} \approx 0$, also $\frac{dH^*}{dRe_\theta} \ll 1$ (weak depend)

$\therefore \quad \frac{dH^*}{dx} \approx 0$

Therefore from K.E.S equ.

$$\frac{\theta}{H^*}\cancel{\frac{dH^*}{dX}}^{\,0} = \frac{2C_D}{H^*} - C_f/2 + (H-1)\frac{\theta}{u_e}\frac{du_e}{dX}$$

$$\therefore \quad \frac{2C_D}{H^*} = C_f/2 - \left(\frac{H-1}{H}\right)\frac{\delta^*}{u_e}\frac{du_e}{dX}$$

$$= C_f/2 + \left(\frac{H-1}{H}\right)\beta\, C_f/2$$

$$= \frac{C_f}{2}\left[1 + \left(\frac{H-1}{H}\right)\beta\right]$$

Using $G-\beta$ equation

$$\frac{2C_D}{H^*} = C_f/2\left[1 + \left(\frac{H-1}{H}\right)\left(\frac{G^2}{A^2B} - \frac{1}{B}\right)\right]$$

$$= C_f/2\left[1 + \left(\frac{H-1}{H}\right)\frac{1}{C_f/2}\left(\frac{H-1}{H}\right)^2 0.003 - \left(\frac{H-1}{H}\right)\frac{1}{0.75}\right] \Big/ \begin{matrix} A = 6.7 \\ B = 0.75 \end{matrix}$$

$$= \frac{C_f}{2}\left[1 - \frac{1}{0.75}\,\frac{H-1}{H}\right] + 0.03\left(\frac{H-1}{H}\right)^3$$

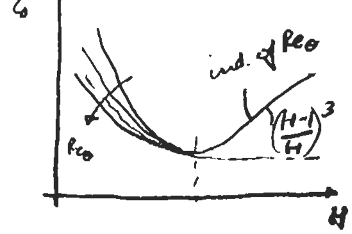
Recall

$$\frac{H-1}{H} = \frac{3}{4}\left(1 - U_s\right)$$

$$\therefore \quad \frac{2C_D}{H^*} = \underbrace{C_f/2 \cdot U_s}_{\substack{\text{shear stress velocity} \\ \text{unit}}} + \underbrace{0.03\left(\frac{H-1}{H}\right)^2}_{\substack{\text{shear stress} \\ \text{order}}}\ \underbrace{\frac{3}{4}\left(1-U_s\right)}_{\text{velocity}}$$

$\uparrow$ $\tau\ \Delta u$ $\qquad\qquad$ $\uparrow$ $\tau\ \Delta u$

fudge factor $\rightarrow F\,C_\tau\,(1-U_s)$

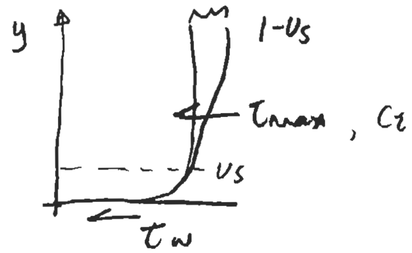


laminar is limit
2 orders of magnitude
in C_D

Non-Equilibrium Effects.

Assume C_D is valid for non-equilibrium flows.

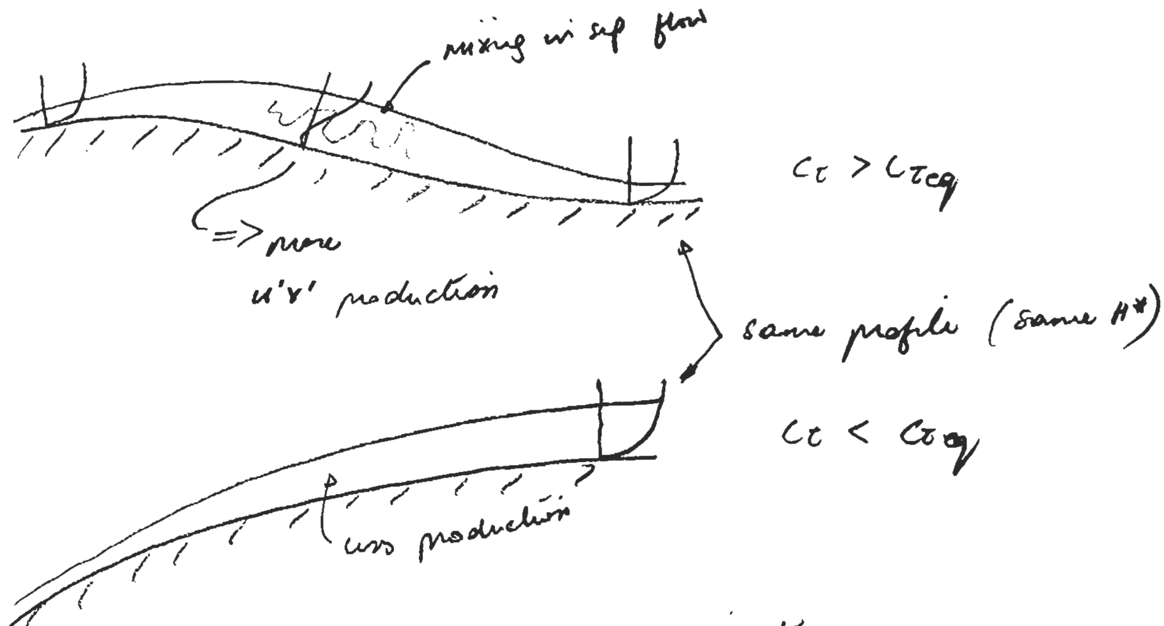
C_D — destruction of K.E. into heat



$$C_\tau = \frac{\pi}{4} K \left(1-U_s\right)^2$$ — local dependance on H.

Thought expt.

a)



$\Rightarrow$ <u>C_D is different</u>

Lag effect: $\overline{u'v'}$ depends not only on local Re_θ, H but also on upstream BL evolution (history)

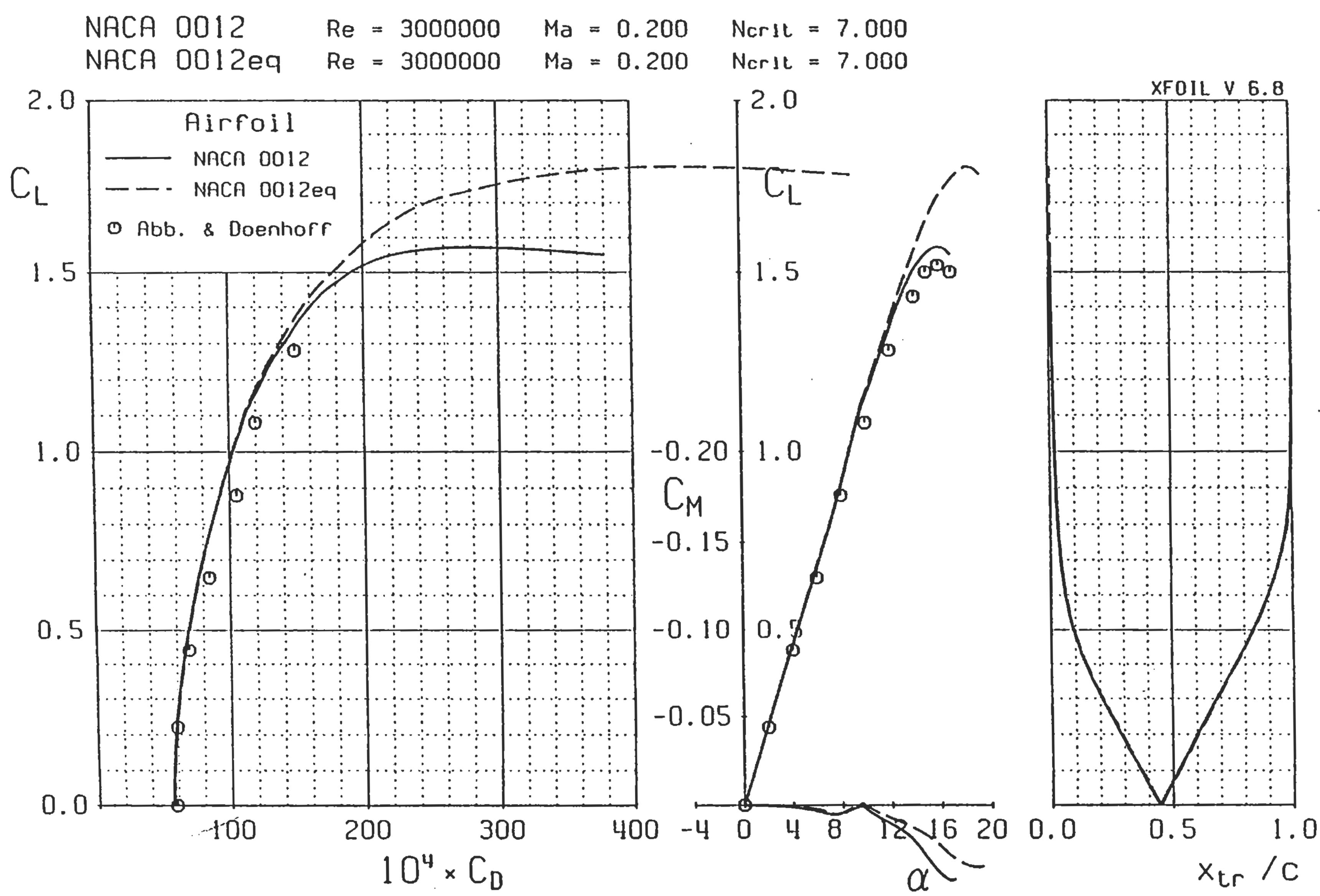
$$\frac{2C_D}{H^*} = \underset{\substack{\uparrow \\ \text{no lag}}}{\frac{C_f}{2} U_s} + \underset{\substack{\uparrow \\ \text{lag effect.}}}{C_\tau(x)\left(1-U_s\right)}$$

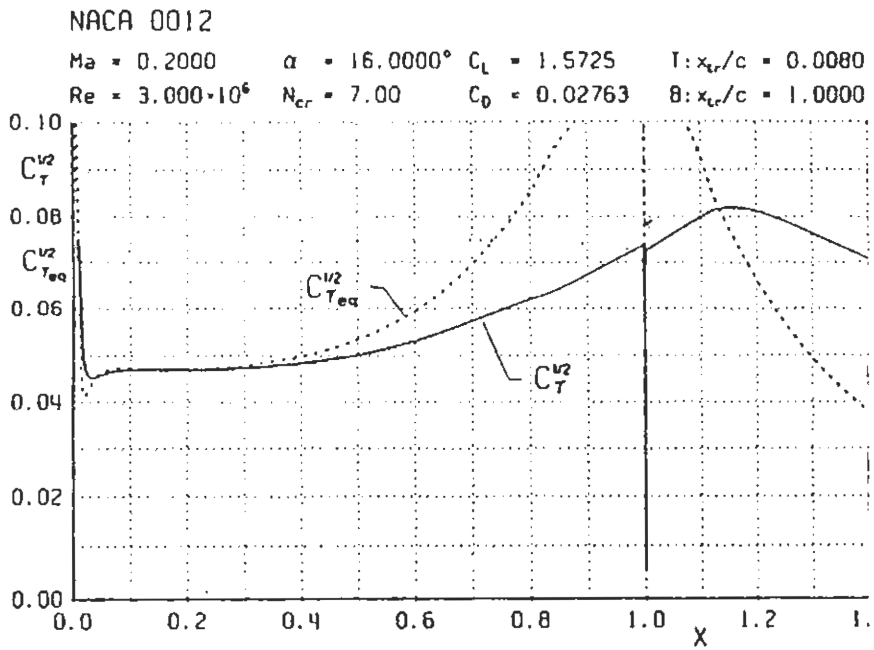
$C_\tau(x)$ is an independant variable ∴ introduce
3^{rd} equation :

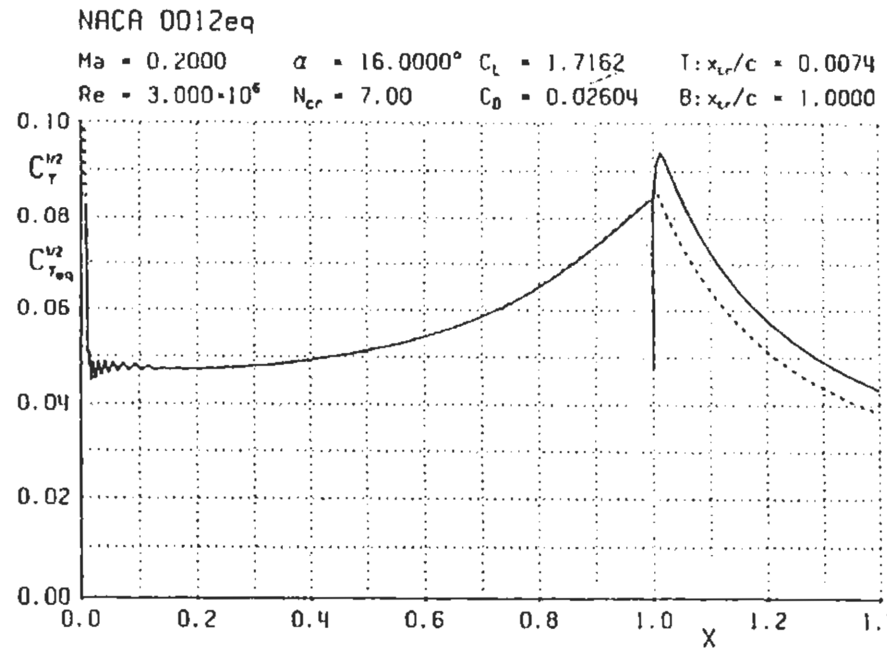
$$\frac{dC_\tau}{dx} = \text{- - - -}$$

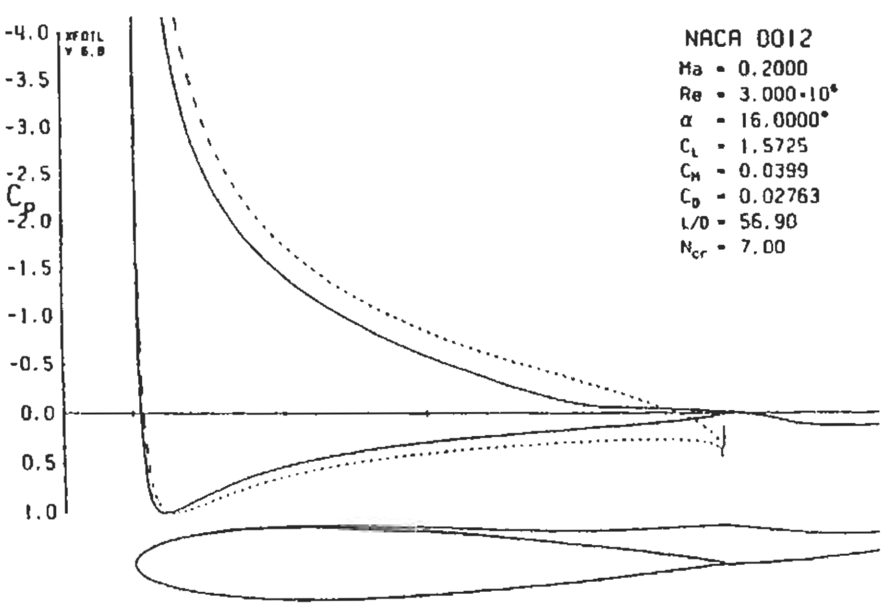
$$\frac{\delta}{C_\tau}\frac{dC_\tau}{dx} = 4.2\left(\sqrt{C_{\tau eq}} - \sqrt{C_\tau}\right)$$

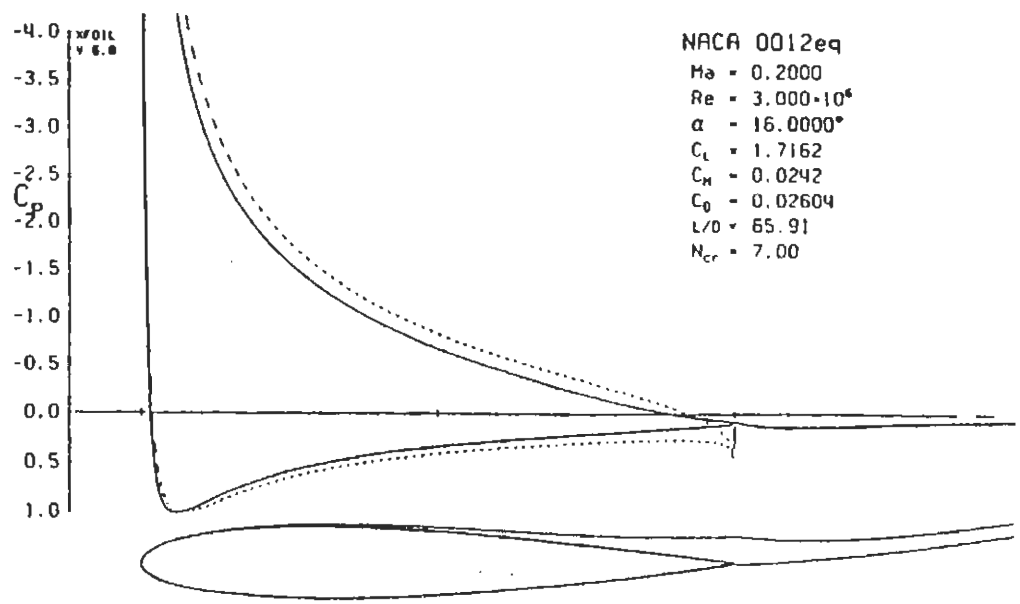
Deviation of turbulent BL from G-β locus is governed by
lag equation


NACA 0012 Re = 3000000 Ma = 0.200 Ncrit = 7.000
NACA 0012eq Re = 3000000 Ma = 0.200 Ncrit = 7.000
XFOIL V 6.8
Airfoil
NACA 0012
NACA 0012eq
Abb. & Doenhoff
C_L
C_M
2.0
1.5
1.0
0.5
0.0
-0.20
-0.15
-0.10
-0.05
100 200 300 400
$10^4 \times C_D$
-4 0 4 8 12 16 20
α
0.0 0.5 1.0
x_{tr} /c


NACA 0012
Ma = 0.2000 α = 16.0000° C_L = 1.5725 T: x_tr/c = 0.0080
Re = 3.000·10^6 N_cr = 7.00 C_D = 0.02763 B: x_tr/c = 1.0000
C_T^1/2
C_Teq^1/2
0.10
0.08
0.06
0.04
0.02
0.00
0.0 0.2 0.4 0.6 0.8 1.0 1.2
X


NACA 0012eq
Ma = 0.2000 α = 16.0000° C_L = 1.7162 T: x_tr/c = 0.0074
Re = 3.000·10^6 N_cr = 7.00 C_D = 0.02604 B: x_tr/c = 1.0000
C_T^1/2
C_Teq^1/2
0.10
0.08
0.06
0.04
0.02
0.00
0.0 0.2 0.4 0.6 0.8 1.0 1.2 1.4
X


XFOIL V 6.8
C_p
-4.0 -3.5 -3.0 -2.5 -2.0 -1.5 -1.0 -0.5 0.0 0.5 1.0
NACA 0012
Ma = 0.2000
Re = 3.000·10^6
α = 16.0000°
C_L = 1.5725
C_M = 0.0399
C_D = 0.02763
L/D = 56.90
N_cr = 7.00


XFOIL V 6.8
C_p
-4.0 -3.5 -3.0 -2.5 -2.0 -1.5 -1.0 -0.5 0.0 0.5 1.0
NACA 0012eq
Ma = 0.2000
Re = 3.000·10^6
α = 16.0000°
C_L = 1.7162
C_M = 0.0242
C_D = 0.02604
L/D = 65.91
N_cr = 7.00