Lecture 1   Fall, 03   Sept 3, 03.   (1)

Rm 33-319.
MWF – 9–10

A> Course Admin
B> Topics / Context
C> Kinematic Components

A> Admin
- MWF 9–10
- No Quizzes, No Final
- Grading on P.Sets – 7–8, 2 major ones
- Group discussion O.K
- Refs for class

B> Topics / Context put in   Readings: Batch 78–87, White 16–22

Look at flow regimes using non-dim. parameters

a> $Kn = \lambda / L$ λ = mean free path
L = characteristic length

$Kn \gg 1 \Rightarrow$ Discrete flow, rarified

$Kn \ll 1 \Rightarrow$ <u>Continuum</u> flow

b> $Ma = V/c$ $Ma \ll 1 \Rightarrow$ Incompressible, $\rho \sim$ const
$Ma \approx 1 \Rightarrow$ Compressible, ρ is variable

Focus on incompressible flows + compressibility correction

c> $k = \frac{\omega L}{V}$ = non-dimension or reduced frequency.

$k \ll 1 \Rightarrow$ <u>steady</u> flow

d) $Re = \frac{VL}{\nu}$ $\nu = \mu/\rho$ $Re = \frac{\text{dynamic / momentum flux}}{\text{shear stress}}$ ②

$$\sim \frac{\rho V^2}{\mu\left(\frac{V}{L}\right)}$$

$Re << 1$ — Stokes flow

$\simeq 1$ — Oseen flow

$>> 1$ — High Reynolds # flow
(thin shear + inviscid outer flow)

∞ — Inviscid

From kinetic theory

$\mu \sim 1/2 \rho \bar{a} \lambda$ $\bar{a} \sim c$ (speed of sound)

$$Re \sim \frac{\rho V L}{1/2 \rho c \lambda} \sim M\left(\frac{L}{\lambda}\right) >> 1$$

$$\therefore \frac{M}{Re} \sim \frac{\lambda}{L} << 1$$

$M << Re$ (low Re, high/low Mach, continuum assumptions breaks down)

In addition

$\delta \sim$ B.L thickness

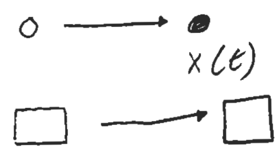
$$\frac{\lambda}{\delta} \sim \frac{\lambda}{L} \cdot \frac{L}{\delta} << 1 \qquad \frac{L}{\delta} \sim \sqrt{Re}$$

$$\therefore \frac{M}{Re}\sqrt{Re} << 1 \Rightarrow \frac{M}{\sqrt{Re}} << 1 \text{ thin layer}$$

c> Kinematic ~~Components~~

a> Hierarchy of Kinematics - Increasing level of complexity of motion

1> Point-mass motion:
(rigid-body translation)

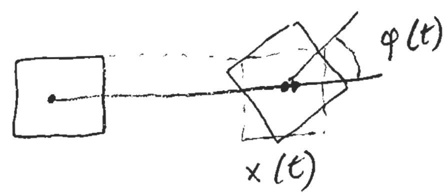


velocity $v(t) = \dot{x}(t)$

acceleration $a(t) = \dot{v}(t) = \ddot{x}(t)$

2> Rigid-body motion:

translation + rotation

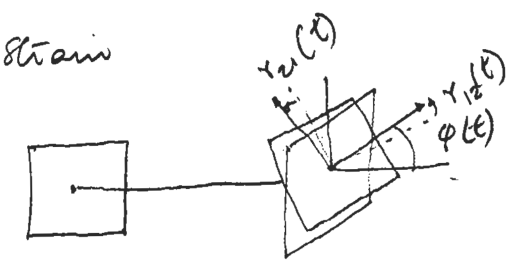


additional: $\Omega = \dot{\varphi}(t)$

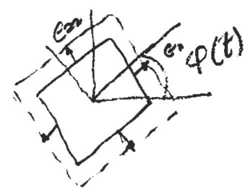
$\alpha = \dot{\Omega}(t)$

3> Deformable-body motion:

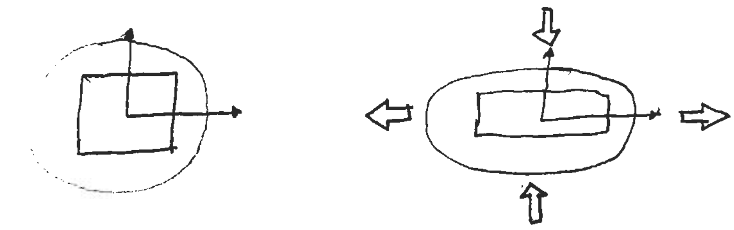
trans. + rotation + strain



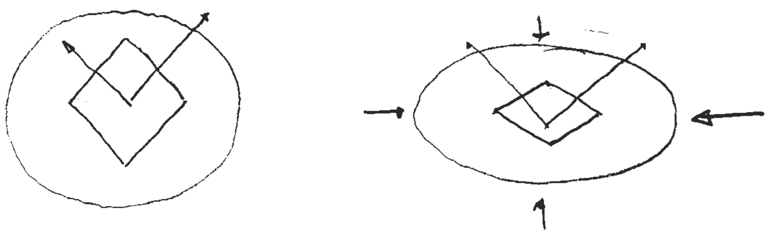
shear strain



strain tensor : $\begin{bmatrix} \epsilon_{11} & \gamma_{12} & \gamma_{13} \\ \gamma_{21} & \epsilon_{22} & \gamma_{23} \\ \gamma_{31} & \gamma_{32} & \epsilon_{33} \end{bmatrix}$



$\epsilon \neq 0 \quad \gamma = 0$

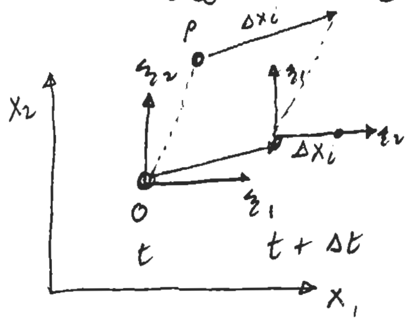


$\epsilon = 0 \, , \, \gamma \neq 0$

Whether a strain is a shear or a normal strain depends on orientation of reference axis

c) i) Kinematic Components (convection + vorticity + strain rate)

Examine linear displacements of two points O and P attached to material. Define material axes ξ_i



$$x_i(\xi_i ; t)$$

$$\Delta x_i^P = \Delta x_i^O + \frac{\partial \Delta x_i^P}{\partial \xi_i} \xi_j + HOT$$

$$a_{ij} = \frac{\partial \Delta x_i}{\partial \xi_j} \quad (\text{drop } P)$$

Useful to write as $= \dfrac{a_{ij} - a_{ji}}{2} + \dfrac{a_{ij} + a_{ji}}{2}$

$$a_{ij} = \varphi_{ij} + s_{ij}$$

anti sym symmetric

$$\therefore \quad \Delta x_i^P = \Delta x_i^O + \varphi_{ij}\xi_j + s_{ij}\xi_j$$

In vector notation $\overrightarrow{\Delta X}^P = \overrightarrow{\Delta X}^O + \bar{\bar{a}} \cdot \vec{\xi}$, where $\bar{\bar{a}} = \bar{\bar{\varphi}} + \bar{\bar{s}}$

Introduce time dependance

$$\Delta x_i = u_i \Delta t$$

$$\varphi_{ij} = \tfrac{1}{2}\omega_{ij}\Delta t$$

$$s_{ij} = e_{ij}\Delta t$$

$$\Rightarrow \quad u_i^P = u_i^O + \tfrac{1}{2}\omega_{ij}\xi_j + e_{ij}\xi_j$$

$$\frac{\partial u_i}{\partial \xi_j} = \tfrac{1}{2}\,\omega_{ij} + e_{ij} \quad , \quad \text{where} \quad \omega_{ij} \equiv \frac{\partial u_i}{\partial \xi_j} - \frac{\partial u_j}{\partial \xi_i} \qquad e_{ij} \equiv \tfrac{1}{2}\left(\frac{\partial u_i}{\partial \xi_j} + \frac{\partial u_j}{\partial \xi_i}\right)$$

angular rotation

$\Leftarrow$ follows from φ & S

$$\frac{\partial u_i}{\partial \xi_j} = \nabla \vec{u} = \begin{bmatrix} & 3\times 3 & \end{bmatrix} \qquad \bar{\bar{\omega}} = \begin{bmatrix} 0 & \cdot & \cdot \\ - & 0 & \cdot \\ - & - & 0 \end{bmatrix} \text{ - antisymmetric}$$

$$\bar{\bar{e}} = \begin{bmatrix} \cdot & + & + \\ + & \cdot & + \\ + & + & \cdot \end{bmatrix} \text{ - symmetric}$$

$$\bar{\bar{\omega}} = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & \omega_x \\ -\omega_y & -\omega_x & 0 \end{bmatrix} \quad , \quad \bar{\bar{\omega}} \cdot \vec{\xi} = \vec{\omega} \times \vec{\xi} \ ,$$

In 2D.

$$\omega_x = \omega_y = 0 \qquad \omega_z = \frac{\partial v_2}{\partial \xi_1} - \frac{\partial u_1}{\partial \xi_2}$$

$$\therefore \ \nabla \vec{u} = \begin{bmatrix} \frac{\partial u_1}{\partial \xi_1} & \frac{\partial u_1}{\partial \xi_2} \\ \frac{\partial u_2}{\partial \xi_1} & \frac{\partial u_2}{\partial \xi_2} \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 0 & -\left(\frac{\partial u_2}{\partial \xi_1} - \frac{\partial u_1}{\partial \xi_2}\right) \\ \left(\frac{\partial u_2}{\partial \xi_1} - \frac{\partial u_1}{\partial \xi_2}\right) & 0 \end{bmatrix}$$

$$+ \begin{bmatrix} \frac{\partial u_1}{\partial \xi_1} & \tfrac{1}{2}\left(\frac{\partial u_1}{\partial \xi_2} + \frac{\partial u_2}{\partial \xi_1}\right) \\ \tfrac{1}{2}\left(\frac{\partial u_1}{\partial \xi_2} + \frac{\partial u_2}{\partial \xi_1}\right) & \partial u_2/\partial \xi_2 \end{bmatrix}$$

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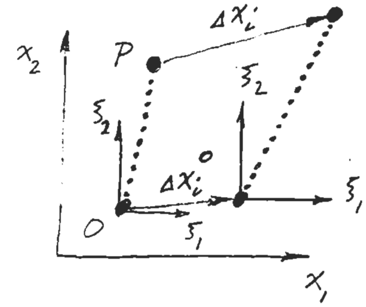
Kinematic Components

Linear displacements of two points O and P are related by

local coordinate

$$\Delta x_i = \Delta x_i^o + \frac{\partial(\Delta x_i)}{\partial \xi_j}\,\xi_j \qquad \xi_i = \overrightarrow{PO}$$



define $\dfrac{\partial(\Delta x_i)}{\partial \xi_j} = a_{ij}$ displacement-gradient tensor

symmetric   antisymmetric

$$s_{ij} = \frac{a_{ij} + a_{ji}}{2} \qquad \varphi_{ij} = \frac{a_{ij} - a_{ji}}{2}$$

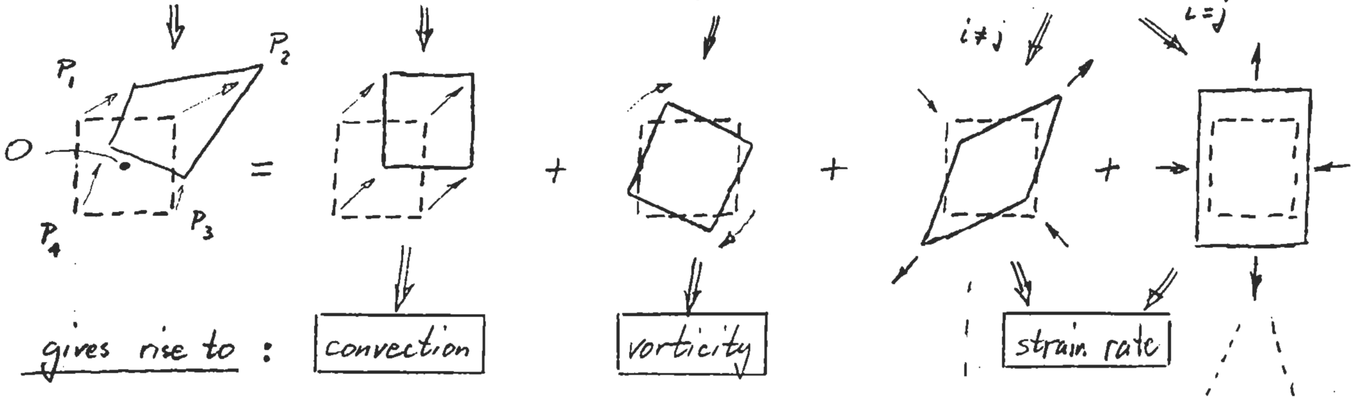
so that $a_{ij} = s_{ij} + \varphi_{ij}$

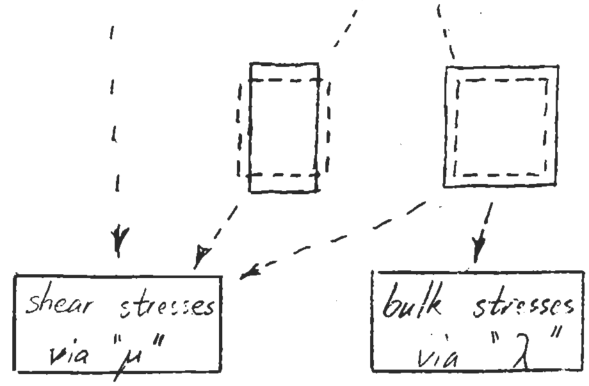
$$\Delta x_i = \Delta x_i^o + \varphi_{ij}\,\xi_j + s_{ij}\,\xi_j$$

Introduce time dependence: $\Delta x_i = u_i\,\Delta t$, $\varphi_{ij} = \frac{1}{2}\omega_{ij}\,\Delta t$, $s_{ij} = e_{ij}\,\Delta t$

motion of "O"   antisymm. motion about "O"   symm. motion about "O"

$$u_i = u_i^o + \frac{1}{2}\omega_{ij}\,\xi_j + e_{ij}\,\xi_j$$





All these components are related to the velocity field