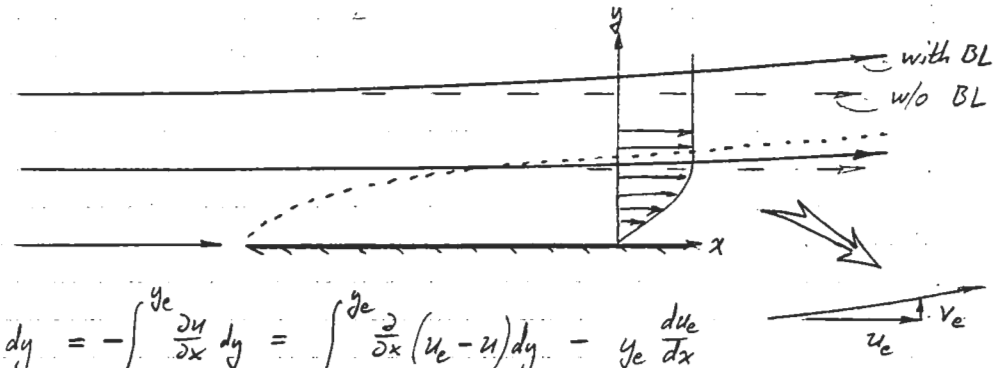
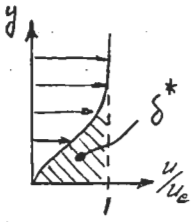
Displacement Effects of Boundary Layer on Potential Flow

Actual Flow



$$v_e(x) \equiv v(x, y_e) = \int_0^{y_e} \frac{\partial v}{\partial y}\, dy = -\int_0^{y_e} \frac{\partial u}{\partial x}\, dy = \int_0^{y_e} \frac{\partial}{\partial x}(u_e - u)\, dy - y_e \frac{du_e}{dx}$$

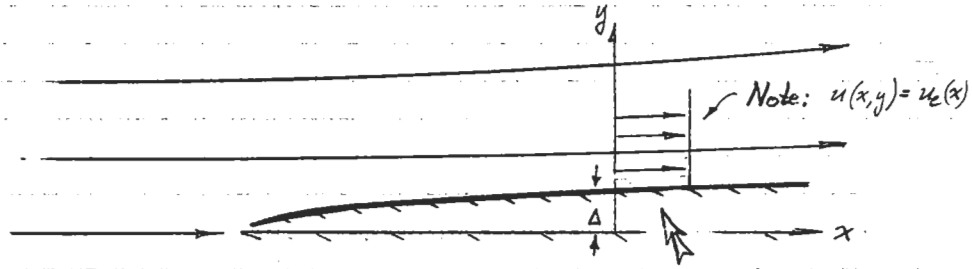
$$= \frac{d}{dx}\left[u_e \int_0^{y_e} \left(1 - \frac{u}{u_e}\right) dy\right] - y_e \frac{du_e}{dx}$$



or $$\boxed{v_e = \frac{d}{dx}(u_e \delta^*) - y_e \frac{du_e}{dx}}$$ where $$\delta^* = \int_0^{y_e} \left(1 - \frac{u}{u_e}\right) dy$$

(displacement thickness)

Displacement Body Model

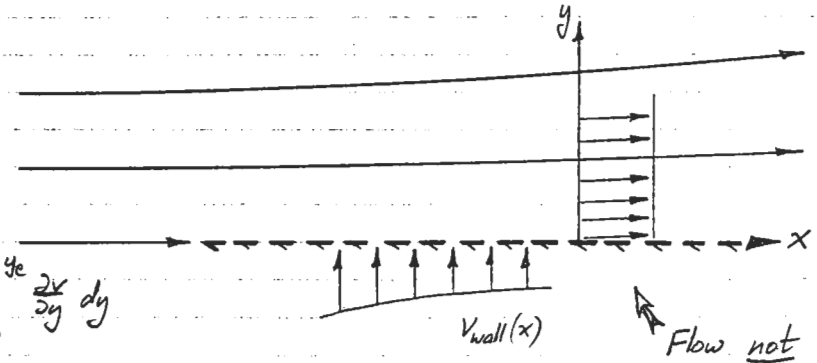


$$v_e = u_e \frac{d\Delta}{dx} + \int_\Delta^{y_e} \frac{\partial v}{\partial y}\, dy$$

$$= u_e \frac{d\Delta}{dx} - \int_\Delta^{y_e} \frac{\partial u}{\partial x}\, dy = u_e \frac{d\Delta}{dx} - (y_e - \Delta)\frac{du_e}{dx}$$

or $$\boxed{v_e = \frac{d}{dx}(u_e \Delta) - y_e \frac{du_e}{dx}}$$ $\Rightarrow$ $\underline{\Delta = \delta^*}$ (by comparing with Actual Flow v_e)

Wall Blowing Model



$$v_e = v_{wall} + \int_0^{y_e} \frac{\partial v}{\partial y}\, dy$$

$$= v_{wall} - \int_0^{y_e} \frac{\partial u}{\partial x}$$

or $$\boxed{v_e = v_{wall} - y_e \frac{du_e}{dx}}$$ $\Rightarrow$ $\underline{v_{wall} = \frac{d}{dx}(u_e \delta^*)}$ (by comparing with Actual Flow)