1a>

7 scales : $c, b, t, \alpha, U_\infty, \nu, \rho$

− 3 units : M, L, T

= 4 non-dimension parameters :

b/c = wing aspect ratio

α = wing angle of attack

t/c : thickness ratio

$\frac{U_\infty c}{\nu}$ = chord Reynolds number

Other combinations are possible (ex. t/b, $U_\infty b/\nu$), but these may not be relevant or useful.

1b> Non-dimensional variables:

$\vec{x}^* = \vec{x}/c \qquad \vec{u}^* = \vec{u}/U_\infty$

$p^* = P/\rho U_\infty^2 \qquad$ etc.

Continuity :

$$\nabla^* \cdot \vec{u}^* = 0 \quad \text{where} \quad \nabla^* = \hat{i}\frac{\partial}{\partial x^*} + \hat{j}\frac{\partial}{\partial y^*} + \hat{k}\frac{\partial}{\partial z^*}$$

Momentum :

$$(\vec{u}^* \cdot \nabla^* \vec{u}^*) = -\nabla^* p^* + \frac{1}{Re}\nabla^{*2}\vec{u}^*$$

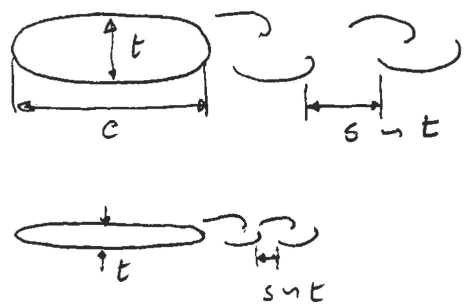
where

$$\nabla^{*2} = \frac{\partial^2}{\partial x^{*2}} + \frac{\partial^2}{\partial y^{*2}} + \frac{\partial}{\partial z^{*2}}$$

$$Re = \frac{U_\infty c}{\nu}$$

The other parameters b/c, t/c, α would enter in boundary conditions, i.e. no-slip BC $\vec{u}^* = 0$ on the wing surface (or slip $\vec{u}^* \cdot \hat{n}_{wing} = 0$), and/or farfield.

1c)



streamwise spacing of shed vortices is most likely proportional to t. There may be some influence of c (chord)

The vortices move downstream at some speed proportional to V_∞. The shedding frequency is proportional to

$$f \sim \frac{U}{s} \sim \frac{U}{t} \quad \text{(vortices/unit time)}$$

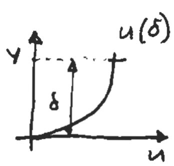
$$\longrightarrow \quad St = ft/V_\infty \quad \text{(Strouhal number)}$$

This will depend mainly on t/c and somewhat on Re.

2a) The overall flow <u>does not</u> have any geometric length scale. $l = \nu/U$ is the only length scale available to non-dimensionalize the problem. No non-dim. parameters such as $c/l = Uc/\nu = Re_c$ exist.

2b) The non-dimensional form $\delta/L = f(X/L)$ gives the geometry $\delta(x)$ of <u>any</u> Blasius boundary layer, since there are no other non-dimensional parameters in the problem.

$$\delta/l = C\sqrt{\frac{x}{L}} \quad \text{where } C \approx 5.0 \text{ for } u(\delta) \simeq 0.99\,V_\infty$$



3) X-momentum and continuity ...

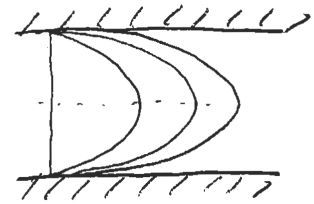
$$\frac{\partial u}{\partial t} + \cancel{u\frac{\partial u}{\partial x}} + \cancel{v\frac{\partial u}{\partial y}} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\cancel{\frac{\partial^2 u}{\partial x^2}} + \frac{\partial^2 u}{\partial y^2}\right)$$

$$\Rightarrow \frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2}$$

$\therefore$ Steady state solution is: $u(y) = \frac{-h^2}{8\mu}\left(\frac{\partial p}{\partial x}\right)\left(1 - \left(\frac{2y}{h}\right)^2\right)$

2a) $\frac{\partial p}{\partial x}$ changes very slowly: dominant balance: $0 \simeq -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2}$

$\Rightarrow$ velocity profile stays parabolic (quasi-steady)

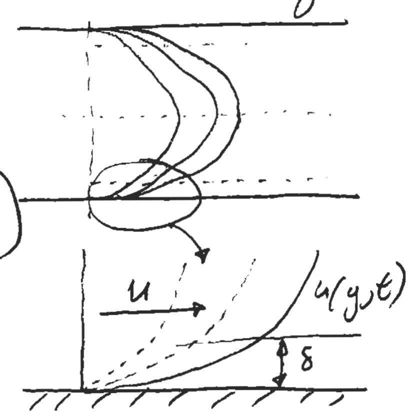


2b) $\frac{\partial p}{\partial x}$ changes very quickly: dominant balance is

$$\frac{\partial u}{\partial t} \simeq -\frac{1}{\rho}\frac{\partial p}{\partial x}$$

except near the wall, where $\partial u/\partial t \to 0$. This produces a uniform change in $u(y)$ except near the wall

We can estimate the thickness of the boundary layer after some short time t:



Outside BL: $\frac{\partial^2 U}{\partial t^2} \simeq -\frac{1}{\rho}\frac{\partial}{\partial t}\left(\frac{\partial p}{\partial x}\right) \longrightarrow U \simeq \frac{1}{2}t^2\left(-\frac{1}{\rho}\frac{\partial}{\partial t}\left(\frac{\partial p}{\partial x}\right)\right)$

Order estimate inside BL: $\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2}$

$$\frac{U}{t} \sim -\frac{1}{\rho}\frac{\partial}{\partial t}\left(\frac{\partial p}{\partial x}\right)\cdot t \sim \nu\frac{U}{\delta^2}$$

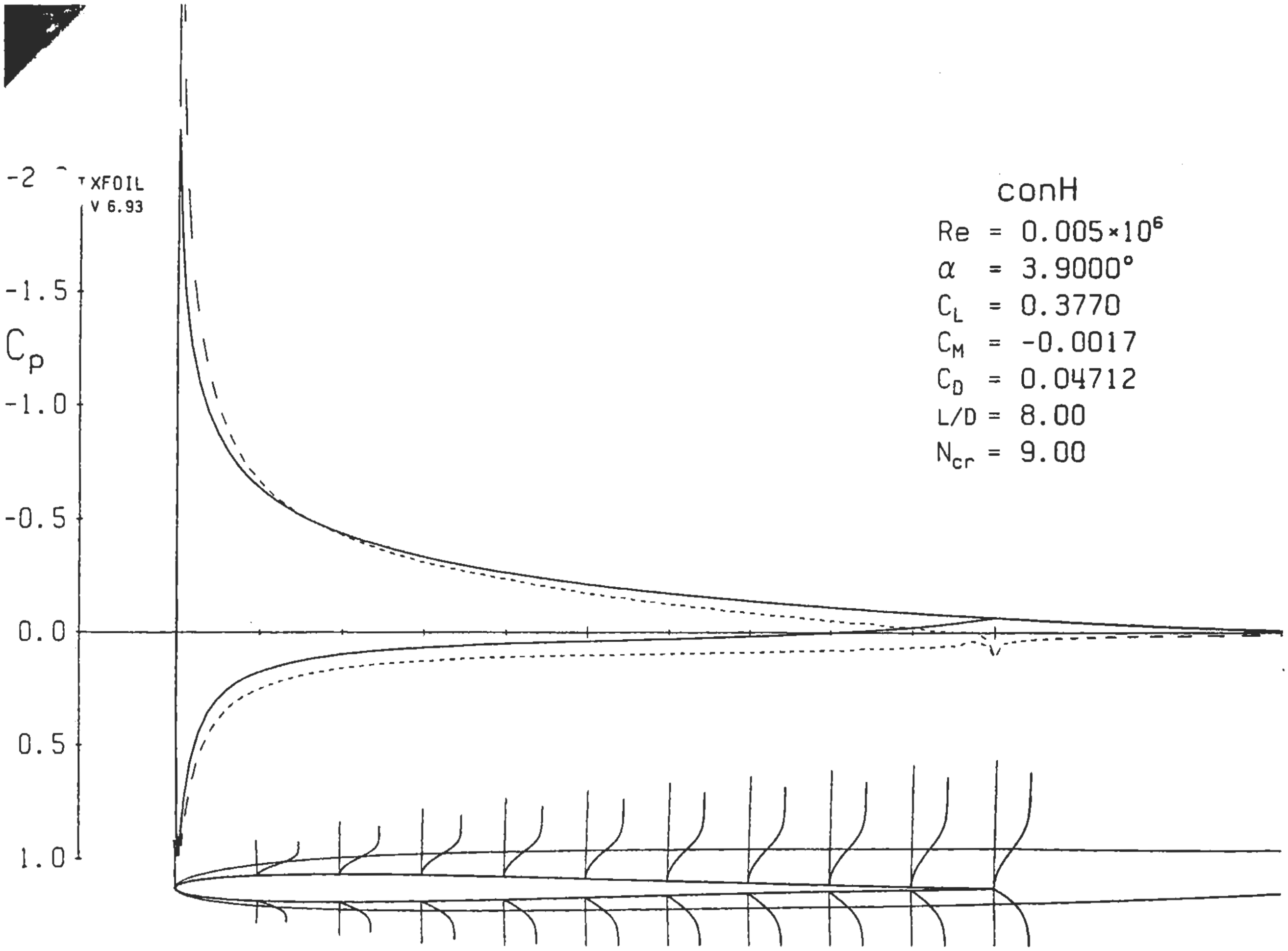
$\longrightarrow \delta = O\left(\sqrt{\nu t}\right) \longrightarrow$ same as Rayleigh problem

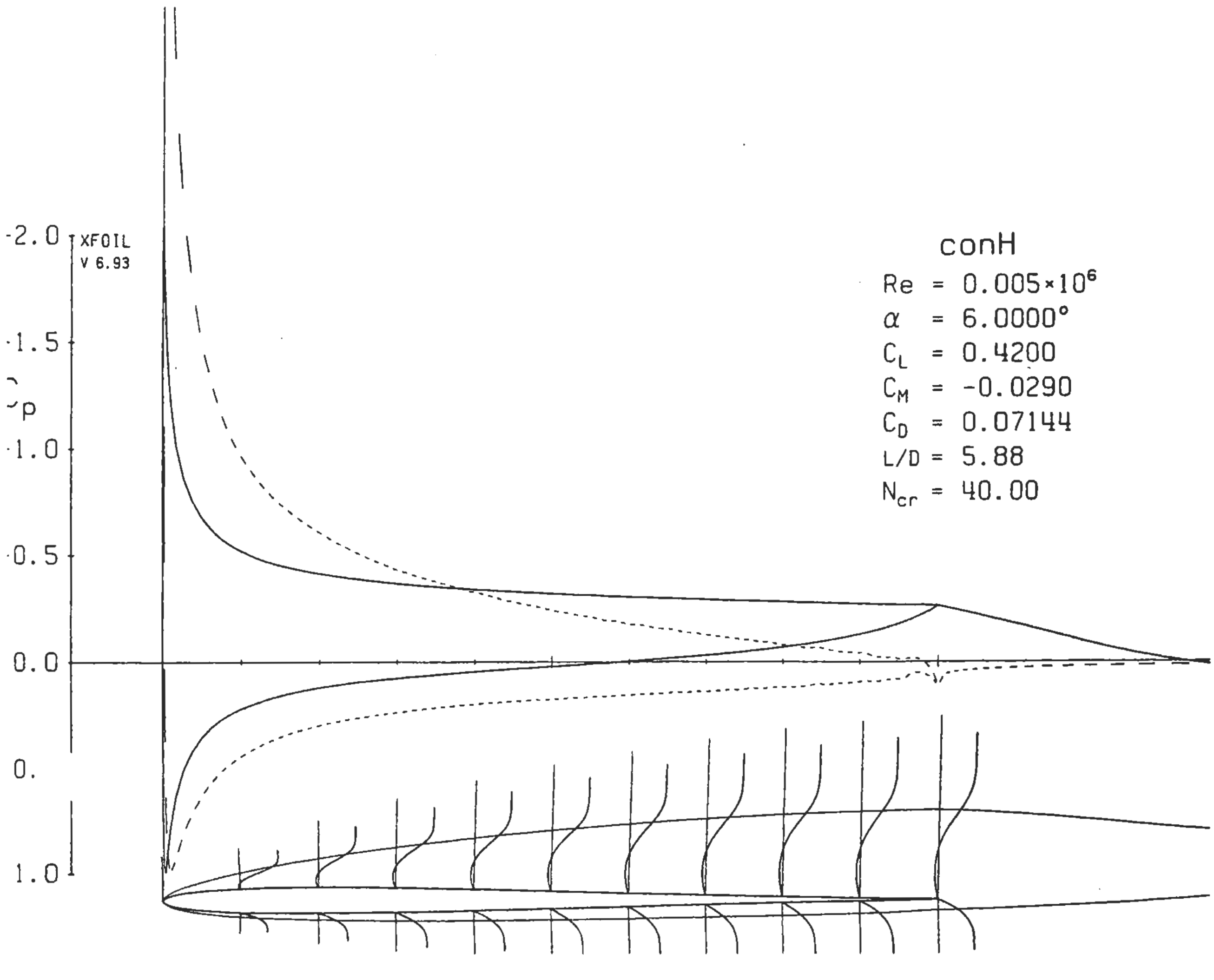
2c) Change over from slow to fast is when $\delta = O(h)$, i.e. the time it takes for the boundary layer to fill up the channel so that $u(y)$ is parabolic

At the centerline: $u = -\frac{h^2}{8\mu}\frac{\partial p}{\partial x} \simeq \frac{1}{2}t^2\left(-\frac{1}{\rho}\frac{\partial}{\partial t}\left(\frac{\partial p}{\partial x}\right)\right) \to t^2 \simeq \frac{h^2}{4\nu}\frac{\partial p/\partial x}{\partial/\partial t(\partial p/\partial x)}$

$$\Rightarrow \frac{\partial}{\partial t}\left(\frac{\partial p}{\partial x}\right) \simeq \frac{\nu}{4h^2}\left(\frac{\partial p}{\partial x}\right) \quad \text{since } t^2 \simeq \frac{h^4}{\nu^2}$$

or, $\frac{\partial}{\partial t}\left(\frac{\partial p}{\partial x}\right) \simeq 2\left(\frac{\mu^2}{\rho}\frac{U_{center}}{h^4}\right) \Rightarrow$ U at the centerline roughly doubles at t change over.


XFOIL
V 6.93
-2
-1.5
C_p
-1.0
-0.5
0.0
0.5
1.0
conH
Re = 0.005×10^6
α = 3.9000°
C_L = 0.3770
C_M = -0.0017
C_D = 0.04712
L/D = 8.00
N_{cr} = 9.00


-2.0
XFOIL
V 6.93
-1.5
-1.0
-0.5
0.0
0.
1.0
conH
Re = 0.005×10^6
α = 6.0000°
C_L = 0.4200
C_M = -0.0290
C_D = 0.07144
L/D = 5.88
N_{cr} = 40.00

1a) $C_p \equiv \dfrac{p - p_\infty}{\frac{1}{2}\rho U_\infty^2}$, $p + \frac{1}{2}\rho U_e^2 = p_\infty + \frac{1}{2}\rho U_\infty^2 \rightarrow C_p = 1 - \left(\dfrac{U_e}{U_\infty}\right)^2$

Given $(u_e/U_\infty) \simeq (x/c)^{\pm a} \Rightarrow C_p = 1 - (x/c)^{\pm 2a}$

$$C_L = L/\tfrac{1}{2}\rho U_\infty^2 c = \frac{1}{\frac{1}{2}\rho U_\infty^2 c}\int_0^1 (p_L - p_u)\,dx = \int_0^1 (C_{p_L} - C_{p_u})\,d(x/c) = \frac{1}{1-2a} - \frac{1}{1+2a} = \underline{\underline{\frac{4a}{1-4a}}}$$

1b) Max. C_L will occur when upper surface is at separation, which corresponds to $a = 0.0904 \rightarrow C_L = \underline{\underline{0.374}}$. This is independant of Re, since laminar separation is independant of Reynolds number.

1c) $\dfrac{\theta(x)}{c} = \theta_1\sqrt{\dfrac{\nu x}{u_e c^2}} = \underline{\underline{\dfrac{\theta_1}{\sqrt{Re_c}}\left(x/c\right)^{(1-m)/2}}}$ Top Surf: $m = -0.0904$, $\theta_1 = 0.868$; Bot Surf: $m = +0.0904$, $\theta_1 = 0.567$

$C_f(x) = 2\sqrt{\dfrac{\nu}{u_e x}}\cdot f_0'' = 2f_0''\dfrac{1}{\sqrt{Re_c}}\left(x/c\right)^{-(1+m)/2}$ Top Surf: $m = -0.0904$, $f_0'' = 0$; Bot Surf: $m = 0.0904$, $f_0'' \approx 0.48$

$$C_{D_{friction}} = \frac{1}{\frac{1}{2}\rho U_\infty^2 c}\int_0^c (\cancel{\tau_{w_u}}^{\,0} + \tau_{w_L})\,dx = 2f_0''\frac{1}{\sqrt{Re_c}}\int_0^1 (x/c)^{2a-(1+a)/2} = \frac{1.51}{\sqrt{Re_c}}$$
$$= \int_0^1 C_{f_L}\left(\frac{U_e}{U_\infty}\right)^2 d(x/c)$$

$$C_{d_{PROF}} = C_{D_{FRICTION}} + C_{PRESSURE} = \left.\frac{\rho U_e^2\theta}{\frac{1}{2}\rho U_\infty^2 c}\right|_{trailing\ edge} = 2\left[(\theta/c)_u + (\theta/c)_L\right]_{trailing\ edge}$$
$$= \frac{2}{\sqrt{Re_c}}\left[0.868 + 0.567\right] = \underline{\frac{2.87}{\sqrt{Re_c}}}$$

For $Re_c = 5000$, $C_{D_{FRICTION}} = 0.0213$ $C_L/C_{D_F} = 17.5$ (friction only)

$C_{D_{PROFILE}} = 0.0406$ $C_L/C_D = 9.2$ (with pressure)

$C_L/C_D \sim \sqrt{Re_c}$. Larger insects have an advantage.

2) We know that (from Falkner - Skan)

$$\Delta = \sqrt{\nu X / U_e} = \text{const.}\, X^{\frac{1-\beta_u}{2}}$$ can produce similarity

Test if $\Delta \sim X^{\frac{1-\beta_u}{2}}$ holds for candidate definitions

2a) $\tau_w = \rho \frac{\mu U_e}{X} \cdot S_0 = \text{const.}\, X^{\beta_m + \beta_u - 1} = \text{const.}\, X^{\frac{3\beta_u - 1}{2}}$

$$\therefore \quad \frac{\mu U_e}{\tau_w} \sim X^{\beta_u} \cdot X^{\frac{1-3\beta_u}{2}} \sim X^{\frac{1-\beta_u}{2}}$$ => can produce similarity

2b) $\delta^* = \delta_1^* \sqrt{\frac{\nu X}{U_e}} \sim X^{\frac{1-\beta_u}{2}}$ => can produce similarity

2c) $\theta + \delta^* = (\theta_1 + \delta_1^*) \sqrt{\frac{\nu X}{U_e}} \sim X^{\frac{1-\beta_u}{2}}$ => can produce similarity

2d) from Falkner - Skan solution

$$\eta_{99} = \frac{\delta_{99}}{\sqrt{\nu X / U_e}}$$

$\therefore$ for any given β_u

$$\delta_{99} = \eta_{99} \sqrt{\frac{\nu X}{U_e}} \sim X^{\frac{1-\beta_u}{2}}$$ => can produce similarity

Any quantity which $O\left(\sqrt{\frac{\nu x}{U_e}}\right)$ can also serve as a Δ definition