Higher approximations: Slope of the lift and moment about mid-chord

1 References

Kaplan, C.: "Two dimensional subsonic compressible flow past elliptic cylinders." NACA TR No. 724. (1938).

Garrick, I.E. and Kaplan, C.: "On the flow of a compressible fluid by the hodograph method. I. Unification and extension of present day results." NACA ACR I4C24. (1941).

Kaplan, C.: "The flow of a compressible fluid past a curved surface." NACA ARR 3KD2. (1945).

2 Lesson Notes

Here we quote the result of Kaplan for an elliptic cylinder at an angle of attack. The circulation about the elliptic cylinder is so adjusted that the rear extremity of the major axis of the ellipse is a stagnation point.

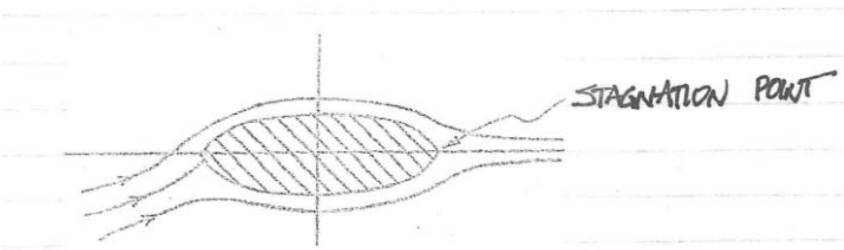

Figure 2.1: Stagnation point

The ratio of the slopes of the lift curve are:

$$\frac{dC_l}{d\alpha}/\frac{dC_l}{d\alpha}_{M_\infty=0} = \mu + \frac{\delta}{1+\delta}[\mu(\mu-1) + \frac{1}{4}\nabla(\mu^2-1)]$$
$$+ \frac{1}{16}(\frac{\delta}{1+\delta})^2[\frac{1}{3}(\mu^2-1)(\nabla+4)^2$$
$$+ \frac{1}{8}(3-log4)(8(\nabla+2)^2$$
$$+ (\mu^2-1)[\nabla^2 + 2(\nabla+2)(3\nabla+8)]])]$$

Where:

- C_l = Section lift coefficient
- α = Angle of attack
- $\mu^2 = (1-M_\infty^2)^{-1}$
- $\nabla = (\gamma+1)(\mu^2-1)$
- δ = Thickness ratio

The above result is valid up to the third approximation, i.e. ϕ^0, ϕ^1, ϕ^2 results are included.

The radio for the moment coefficient about the mid-chord is given by:

$$C_{m_{1/2}}/(C_{m_{1/2}})_{M_\infty=0} = \mu - \frac{1}{32}\frac{\mu^2-1}{\mu}\frac{\delta^2}{1-\delta^2}[16(\nabla+2)^2$$
$$+ (\mu^2-1)[\nabla^2 + 12(\nabla+2)^2]$$
$$- [8(\nabla+2)^2 + (\mu^2-1)[\nabla^2$$
$$+ 2(\nabla+2)(3\nabla+8]]\,log\frac{\mu}{\delta}]$$

Note as the thickness ratio $\delta \to 0$, then the lift slope increases with free stream mach number as $\mu = (1-M_\infty^2)^{-1/2}$. For small but finite thickness ratios, the lift slope increases at a faster rate than μ. This rate of increase is also a function of δ.

Note the $\delta^2 log\delta$ in the expression for the ratio for the moment coefficient, suggesting singular perturbation methodology.

Computed results for $\frac{dC_l}{d\alpha}/\frac{dC_l}{d\alpha}_{M_\infty=0}$ are shown in the following table.

Ratio of Lift Curve Slopes and Moment Coefficients for Compressible and Incompressible Flows

	(dC_l/d_α) / $(dC_l/d_\alpha)_i$				
M°	***μ***	***δ=0.05***	**δ=0.10**	***δ=.015***	**δ=.020**
0.10	1.0050	1.0053	1.0056	1.0058	1.0060
0.20	1.0206	1.0217	1.0228	1.0238	1.0248
0.30	1.0483	1.0511	1.0539	1.0566	1.0592
0.40	1.0911	1.0972	1.1032	1.1095	1.1152
0.50	1.1547	1.1672	1.1799	1.1926	1.2052
0.60	1.2500	1.2760	1.2337	1.3312	1.3594
0.70	1.4005	1.4600	1.5259	1.5957	1.6677
0.80	1.6667	1.8407	2.0524	2.2901	2.5455
0.90	2.2942	1.3327	4.9304	6.9355	9.2308