Fundamentals of Fluid Mechanics

1 FUNDAMENTALS OF FLUID MECHANICS

1.1 ASSUMPTIONS

1. Fluid is a continuum
2. Fluid is inviscid
3. Fluid is adiabatic
4. Fluid is a perfect gas
5. Fluid is a constant-density fluid
6. Discontinuities (shocks, waves, vortex sheets) are treated as separate and serve as boundaries for continuous portions of the flow

1.2 NOTATION

p = pressure (static)
ρ = density
T = temperature (absolute)
$\overline{Q}$ = velocity vector of fluid particles
$\overline{Q} = U_{\overline{i}} + V_{\overline{j}} + W_{\overline{k}}$
$\overline{F}$ = body force per unit mass
$\overline{F} = \nabla\Omega$
Ω = potential of the force field
Gravity field: $\overline{F} = -g\overline{k}$; $\Omega = -gz$
h = enthalpy per unit mass; $h = e + \frac{p}{\rho}$

V' = control volume
S' = surface surrounding V'
σ = impermeable body
$\overline{n}$ = normal directed into the fluid
R = gas constant
c_p = specific heat at constant pressure
c_v = specific heat at constant volume
$\gamma = c_p/c_v$
e = internal energy per unit mass
s = entropy per unit mass

1.3 CONTINUITY EQUATION

$$\frac{\partial\rho}{\partial t} + \nabla(\rho\overline{Q}) = 0$$

$$\frac{D\rho}{Dt} + \rho\nabla\overline{Q} = 0$$

$$\iiint_{V'} \frac{\partial\rho}{\partial t} dV' + \oiint_{S'+\overline{V}} \rho(\overline{Q}\overline{n}) ds' = 0$$

$$\iiint_{V'} \Big[\frac{\partial\rho}{\partial t} + \nabla(\rho\overline{Q})\Big] dV' = 0$$

1.4 Conservation of Momentum

$$\frac{D\overline{Q}}{Dt} = \overline{F} - \frac{\nabla p}{\rho}$$

$$\sum_i \overline{F_i} = \iiint_{V'} \frac{\partial}{\partial t}(\rho \overline{Q}) dV' + \oiint_{S'+\overline{V}} \rho \overline{Q}(\overline{Q}\overline{n}) ds'$$

1.5 Conservation of Thermodynamic Energy

$$\frac{D}{Dt}\left[e + \frac{Q^2}{2}\right] = -\frac{\nabla \cdot (p\overline{Q})}{\rho} + \overline{F} \cdot \overline{Q}$$

$$\rho \frac{D}{Dt}\left[h + \frac{Q^2}{2}\right] = \frac{\partial p}{\partial t} + \rho \overline{F} \cdot \overline{Q}$$

1.6 Equation of State

$p = R\rho T$ (thermally perfect gas)

$c_p, c_v =$ constants (calorically perfect gas)

2 Pressure distribution and compressibility

2.1 Assumptions

1. Steady flow
2. Inviscid fluid
3. No discontinuities (shocks)
4. Perfect gas
5. One-dimensional motion
6. Adiabatic flow
7. $\overline{F} \equiv 0$
8. Isentropic

2.2 Notation

$(\ \ )_0$ = stagnation conditions, $\overline{Q} = 0$
$(\ \ )_\infty$ = free stream conditions, $\overline{Q} = u_{\overline{c}} = u_\infty \overline{c}$
$(\ \ )$ = conditions on body surface (airfoil)

$$\overline{Q} = u'\overline{i} + u'\overline{j} + \omega \overline{k}$$

$$u' = u_\infty + \gamma u$$

2.3 Energy Equations

$$h = e + \frac{p}{\rho}$$

$$d\left[h + \frac{1}{2}Q^2\right] = 0$$

(Heat content plus kinetic energy is constant)

2.4 Perfect Gas Relations

$$p = \rho R T$$

$$pV = RT$$

$$V \equiv \frac{1}{\rho}$$

Can show, without effort:

$$\rho V^{\gamma} = \text{constant}$$

$$p\left(\frac{1}{\rho}\right)^{\gamma} = \text{constant}$$

$$a^2 = \gamma\frac{p}{\rho}, a = \text{speed of sound}$$

$$Q = \sqrt{2c_p\left(T_0 - T\right)}$$

$$T_0 - T = T_0\left[1 - \frac{T}{T_0}\right] = T_0\left[1 - \left(\frac{p}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right]$$

$$Q = \left\{2c_p T_0\left[1 - \left(\frac{p}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right]\right\}^{\frac{1}{2}}$$

2.5 Mach Number

$$M^2 = \frac{Q^2}{a^2} = \frac{2c_p(T_0 - T)}{\gamma\frac{p}{\rho}} = \frac{2c_p(T_0 - T)}{\gamma R T}$$

$$M^2 = \frac{2c_p}{\gamma(c_p - c_v)}\left(\frac{T_0}{T} - 1\right) = \frac{2}{(\gamma - 1)}\left(\frac{T_0}{T} - 1\right)$$

$$\frac{T_0}{T} = \left[1 + \frac{\gamma - 1}{2}M^2\right] = \beta(\gamma, M)$$

$$\frac{p_0}{p} = \left(\frac{T_0}{T}\right)^{\frac{\gamma}{\gamma-1}} = \beta^{\frac{\gamma}{\gamma-1}}$$

$$\frac{\rho_0}{\rho} = \left(\frac{T_0}{T}\right)^{\frac{1}{\gamma-1}} = \beta^{\frac{1}{\gamma-1}}$$

2.6 Other useful forms, expressions

$$Q^2 = 2c_p(T_0 - T)$$

$$a_0^2 = \gamma\frac{p_0}{\rho_0} = \gamma R T_0$$

$$\frac{Q^2}{a_0^2} = \frac{2c_p}{\gamma R}\left(1-\frac{T}{T_0}\right) = \frac{2}{\gamma-1}\left(1-\frac{T}{T_0}\right)$$

$$\frac{T}{T_0} = 1 - \frac{\gamma-1}{2}\left(\frac{Q}{a_0}\right)^2$$

$$\frac{p}{p_0} = \left[1-\frac{\gamma-1}{2}\left(\frac{Q}{a_0}\right)^2\right]^{\frac{\gamma}{\gamma-1}}$$

$$\frac{\rho}{\rho_0} = \left[1-\frac{\gamma-1}{2}\left(\frac{Q}{a_0}\right)^2\right]^{\frac{1}{\gamma-1}}$$

$$a^2 = a_0^2 - \frac{\gamma-1}{2}Q^2$$

2.7 Pressure, velocity relations in isentropic flow

With some effort, one may show:

$$\frac{p}{p_\infty} = \left[1+\frac{\gamma-1}{2}M_\infty^2\left(1-\frac{Q^2}{u_\infty^2}\right)\right]^{\frac{\gamma}{\gamma-1}}$$

Expanding the right-hand side:

$$\frac{p}{p_\infty} = 1+\frac{\gamma}{2}\left(1-\frac{Q^2}{u_\infty^2}\right)M_\infty^2+\frac{\gamma}{8}\left(1-\frac{Q^2}{u_\infty^2}\right)^2M_\infty^4+\frac{\gamma(2-\gamma)}{48}\left(1-\frac{Q^2}{u_\infty^2}\right)^3M_\infty^6+\frac{\gamma(2-\gamma)(3-2\gamma)}{384}\left(1-\frac{Q^2}{u_\infty^2}\right)^4M_\infty^8+\ldots$$

Obtain an expression for

$$c_p = \frac{p-p_\infty}{\frac{1}{2}\rho_\infty u_\infty^2}$$

Let

$$Q = u_\infty + \gamma V, \quad \frac{\gamma V}{U_\infty} \ll 1$$

Find c_p and discuss its limitations.

3 Similarity of flows

3.1 Requirements for similarity of flows

1. Similarity in boundary geometry
   Boundary of one flow can be made to coincide with that of another if its linear dimensions are multiplied by a constant

2. Dynamic constraint
   Dependent variables of one flow are proportional to those of another at the corresponding points.

Example Problem - Illustration

Consider the dynamics of an incompressible fluid flow with constant.
Equation of incompressibility:

$$\frac{Dp}{Dt} = \frac{\partial \rho}{\partial t} + u_i \frac{\partial \rho}{\partial x_i} = 0$$

Equation of continuity:

$$\frac{\partial u_i}{\partial x_i} = 0$$

Introduce dimensionless variables:

$$u_i^{'} = \frac{u_i}{U}, \quad \rho^{'} = \frac{\rho}{\rho_0}, \quad p^{'} = \frac{p}{\rho_0 U^2}, \quad x_i^{'} = \frac{x_i}{L}, \quad t^{'} = \frac{tU}{L}$$

U, ρ_0, L – reference quantities

3.2 Linear momentum

$$\rho^{'} \left(\frac{\partial}{\partial t} + u_\alpha^{'} \frac{\partial}{\partial x_\alpha^{'}} \right) u_i^{'} = -\frac{\partial p^{'}}{\partial x_i^{'}} + \frac{\rho^{'} L}{U^2} F_i + \frac{\gamma}{UL} \frac{\partial^2}{\partial x_\alpha^{'} \partial x_a^{'}} u_i^{'}$$

$$\frac{\partial \rho^{'}}{\partial t^{'}} + u_\alpha^{'} \frac{\partial \rho^{'}}{\partial x_\alpha^{'}} = 0 \qquad \frac{\partial u_\alpha^{'}}{\partial x_a^{'}} = 0$$

Froude no: $F = \frac{U}{\sqrt{gL}} \longrightarrow \frac{\text{inertia forces}}{\text{gravity force}}$
Reynolds no: $\mathrm{Re} = \frac{UL}{\gamma} \longrightarrow \frac{\text{inertia force}}{\text{viscous force}}$

F and Re must be the same for both flows. This is sufficient for dynamic similarity along with similar boundary geometry.

U, ρ_0, L may be different for both flows.

4 Equations governing irrotational flows of a homentropic gas

For this class of flows the simplification is through the introduction of the velocity potential, ϕ, where

$$\overline{Q} = \nabla \phi$$

or

$$u_i = \frac{\partial \phi}{\partial x_i}$$

and the vorticity is zero: $\overline{\omega} = \nabla \times \overline{Q} = \nabla \times \nabla \phi = 0$ where $\overline{\omega}$ is the vorticity vector.

The unsteady Bernoulli equation may be written, for this class of flows:

$$\frac{\partial \overline{Q}}{\partial t} + \nabla \left(\frac{1}{2} Q^2 \right) - \overline{Q} x \overline{\omega} = -\frac{1}{\rho} \nabla p$$

since, $p = p(\rho)$, $\quad \overline{\omega} = 0$

$$\frac{\partial \overline{Q}}{\partial t} + \nabla\left(\frac{1}{2}Q^2\right) + \frac{1}{\rho}\nabla p = 0$$

or

$$\nabla\left(\frac{\partial \phi}{\partial t} + \frac{1}{2}Q^2 + \int \frac{\partial p}{\rho}\right) = 0$$

therefore

$$\frac{\partial \phi}{\partial t} + \frac{1}{2}Q^2 + \int \frac{dp}{\rho} = f(t)$$

Absorb $f(t)$ into ϕ and obtain

$$\frac{\partial \phi}{\partial t} + \frac{1}{2}Q^2 + \int \frac{dp}{\rho} = \text{constant}$$

Differentiate above equation with respect to time, t:

$$\frac{\partial^2 \phi}{\partial t^2} + \overline{Q} \cdot \frac{\partial \overline{Q}}{\partial t} + a^2 \frac{1}{\rho}\frac{\partial \rho}{\partial t} = 0$$

Expressing the continuity equation in terms of ϕ:

$$\frac{1}{\rho}\frac{\partial p}{\partial t} + \nabla^2 \phi + \frac{1}{\rho}\overline{Q} \cdot \nabla \rho = 0$$

Linear momentum equation rewritten yields

$$\overline{Q} \cdot \frac{1}{\rho}\nabla \rho = \frac{1}{a^2}\overline{Q} \cdot \frac{1}{\rho}\nabla p = \frac{1}{a^2}\overline{Q}\left\{-\frac{\partial \overline{Q}}{\partial t} - (\overline{Q} \cdot \nabla)\overline{Q}\right\}$$

Combining the above three equations yields:

$$\frac{1}{a^2}\frac{\partial^2 \Phi}{\partial t^2} + \frac{2}{a^2}\overline{Q} \cdot \frac{\partial \overline{Q}}{\partial t} = \nabla^2 \phi - \frac{1}{a^2}\overline{Q} \cdot \left[(\overline{Q} \cdot \nabla)\overline{Q}\right]$$

since $u_i = \frac{\partial \Phi}{\partial x_i}$, the above equation may be written:

$$*\left(1 - \frac{u^2}{a^2}\right)\frac{\partial^2 \Phi}{\partial x^2} + \left(1 - \frac{v^2}{a^2}\right)\frac{\partial^2 \Phi}{\partial y^2} + \left(1 - \frac{w^2}{a^2}\right)\frac{\partial^2 \Phi}{\partial z^2} - 2\frac{uv}{a^2}\frac{\partial^2 \Phi}{\partial x \partial y} - 2\frac{vw}{a^2}\frac{\partial^2 \Phi}{\partial y \partial z} - 2\frac{uw}{a^2}\frac{\partial^2 \Phi}{\partial x \partial z} =$$

$$\frac{1}{a^2}\frac{\partial^2 \phi}{\partial t^2} + 2\frac{u}{a^2}\frac{\partial^2 \phi}{\partial x \partial t} + 2\frac{v}{a^2}\frac{\partial^2 \phi}{\partial y \partial t} + 2\frac{w}{a^2}\frac{\partial^2 \phi}{\partial z \partial t}$$

where

$$u = \frac{\partial \Phi}{\partial x} \qquad v = \frac{\partial \Phi}{\partial y} \qquad w = \frac{\partial \Phi}{\partial z}$$

For steady flow of a calorically perfect gas:

$$h_0 = \text{constant}$$

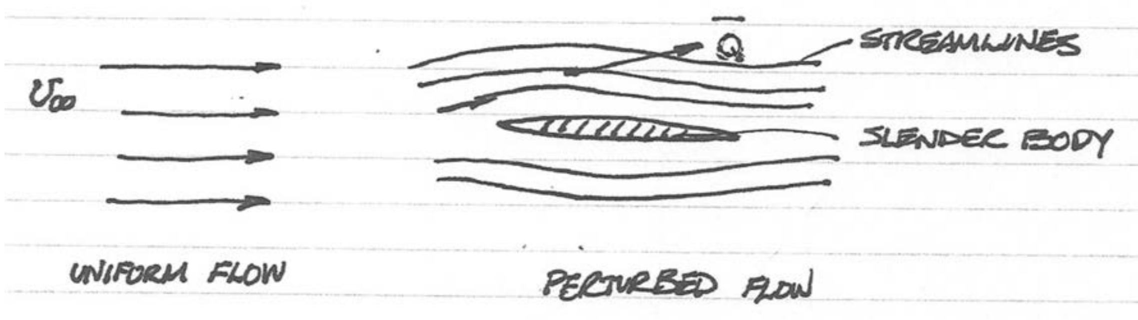
$$c_p T + \frac{Q^2}{2} = c_p T_0$$

$$a^2 = a_0^2 - \frac{\gamma - 1}{2}(\Phi_x^2 + \Phi_y^2 + \Phi_z^2)$$

Equation * is the potential-flow equation.

5 Small Perturbation Theory

We will consider a slender body immersed in a uniform flow, viz.,



in the uniform flow:

$$\overline{Q} = U_\infty \overline{i}$$

in the perturbed flow:

$$\overline{Q} = u_{\overline{i}} + v_{\overline{j}} + w_{\overline{k}}$$

$$\overline{Q} = (U_\infty + u^{'})_{\overline{i}} + v^{'}_{\overline{j}} + w^{'}_{\overline{k}}$$

$$\overline{Q} = \nabla\phi$$

Now define a perturbation velocity potential, $\phi(x, y, z)$, where

$$u^{'} = \frac{\partial\phi}{\partial x}$$

$$v^{'} = \frac{\partial\phi}{\partial y}$$

$$w^{'} = \frac{\partial\phi}{\partial z}$$

$$\therefore \Phi(x, y, z) = U_\infty x + \phi(x, y, z)$$

Using the notation in eqn(*):

$$u = U_\infty + w^{'} = \frac{\partial\Phi}{\partial x} = U_\infty + \frac{\partial\phi}{\partial x}$$

$$v = v^{'} = \frac{\partial\Phi}{\partial y} = \frac{\partial\phi}{\partial y}$$

$$w = w^{'} = \frac{\partial\Phi}{\partial z} = \frac{\partial\phi}{\partial z}$$

$$\Phi_{xx} = \frac{\partial^2\phi}{\partial x^2} = \phi_{xx}$$

$$\Phi_{yy} = \frac{\partial^2\phi}{\partial y^2} = \phi_{yy}$$

$$\Phi_{zz} = \frac{\partial^2\phi}{\partial z^2} = \phi_{zz}$$

$$\Phi_{xy} = \frac{\partial^2\phi}{\partial x \partial y} = \phi_{xy}$$

$$\Phi_{yz} = \frac{\partial^2\phi}{\partial y \partial z} = \phi_{yz}$$

$$\Phi_{xz} = \frac{\partial^2 \phi}{\partial x \partial z} = \phi_{xz}$$

Substituting $\Phi = U_\infty x + \phi$ and multiplying eqn(*) by a^2 we obtain the perturbation equation or perturbation velocity potential equation, for steady flow:

$$**[a^2-(U_\infty+\phi_x)^2]\phi_{xx}+[a^2-(\phi_y)^2]\phi_{yy}+[a^2-(\phi_z)^2]\phi_{zz}-2(U_\infty+\phi_x)\phi_y\phi_{xy}-2(U_\infty+\phi_x)\phi_z\phi_{xz}-2\phi_y\phi_z\phi_{yz}=0$$

Note a^2 may be expressed as:

$$a^2 = a_\infty^2 - \frac{\gamma-1}{2}(2u'U_\infty + u'^2 + v'^2 + w'^2)$$

$$a^2 = a_\infty^2 - \frac{\gamma-1}{2}(2\phi_x U_\infty + (\phi_x)^2 + (\phi_y)^2 + (\phi_z)^2)$$

Also, note that eqn(**) is exact! It is also non-linear.

5.1 Perturbations

Assume the perturbations are small, viz.,

$$\frac{u'}{U_\infty} \ll 1; \qquad \frac{v'}{U_\infty} \ll 1; \qquad \frac{w'}{U_\infty} \ll 1$$

In the limit of small perturbations, we may neglect the terms containing squares of the perturbation velocities in comparison to those containing first powers. Eqn(**) with a^2 substituted becomes

$$(1-M_\infty^2)\phi_{xx}+\phi_{yy}+\phi_{zz} = M_\infty^2(\gamma+1)\frac{\phi_x}{U_\infty}\phi_{xx}+M_\infty^2(\gamma-1)\frac{\phi_x}{U_\infty}(\phi_{yy}+\phi_{zz})+2M_\infty^2\frac{\phi_y}{U_\infty}\phi_{xy}+2M_\infty^2\frac{\phi_z}{U_\infty}\phi_{xz}$$

Note that each term on the right-hand side is non-linear. Each term on the right-hand side contains a perturbation velocity ($\phi_x, \phi_y,$ or ϕ_z). Hence, we may neglect the right-hand side in comparison to the left-hand side. We obtain

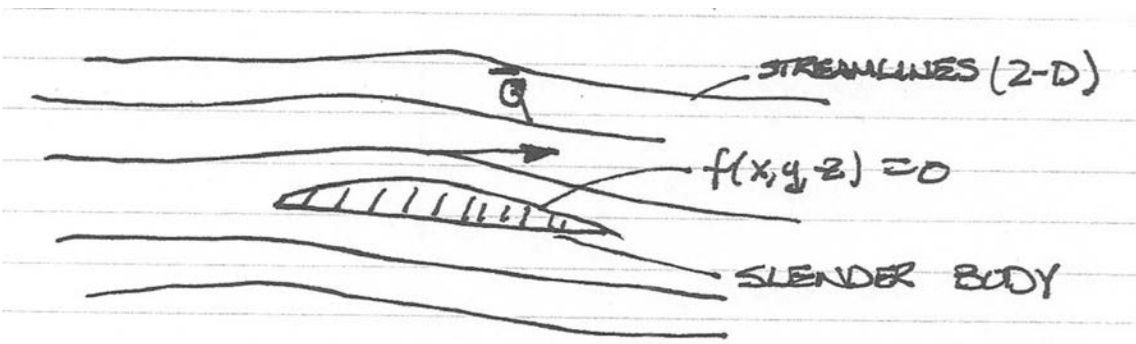
$$(1-M_\infty^2)\phi_{xx}+\phi_{yy}+\phi_{zz}=0$$

5.2 Questions

1. What is the equation where $M_\infty \to 1$?
2. What is the equation where $M_\infty \gg 1$?

6 Boundary Conditions

1. The body surface is a stream line. (inviscid, irrotational flows)
2. Flow velocity must be tangent to body surface
3. Velocity vector has to be orthogonal to the unit normal of the body surface



The body surface is described by $f(x,y,z)$

$$f(x,y,z) = 0$$

Boundary condition is expressed as

$$\overline{Q} \cdot \nabla f(x,y,z) = 0$$

or

$$u_i \frac{\partial f}{\partial x_i} = 0$$

Introducing the perturbation velocities

$$u = U_\infty + u^{'}$$
$$v = v^{'}$$
$$w = w^{'}$$

Substituting,

$$(U_\infty + u^{'})\frac{\partial f}{\partial x} + v\frac{\partial f}{\partial y} + w^{'}\frac{\partial f}{\partial z} = 0$$

Since $u^{'} \ll U_\infty$, we may write:

$$U_\infty \frac{\partial f}{\partial x} + v^{'}\frac{\partial f}{\partial y} + w^{'}\frac{\partial f}{\partial z} = 0$$

This equation must be satisfied on the surface of the body. Consider the two-dimensional case:

$$w^{'} = 0$$
$$\frac{\partial f}{\partial z} = 0$$

We obtain:

$$\frac{v^{'}}{U_\infty} = -\frac{\partial f/\partial x}{\partial f/\partial y} = \frac{dy}{dx}$$

Therefore $\frac{u^{-1}}{U_\infty}$ is the slope of the body (approximately) the slope of the streamline. Recall that

$$u^{'} = \frac{\partial \phi}{\partial y} = U_\infty \frac{dy}{dx}\Big)_{\text{BODY}}$$

Now for thin bodies, a small angle of attack, $y_{\text{BODY}} \approx 0$: this suggests an expansion of $v^{'}(x,y)$ in a powers of y:

$$v^{'}(x,y) = v^{'}(x,0) + \left(\frac{\partial v^{'}}{\partial y}\right)_{y=0} y + \ldots$$

$$\therefore \quad v(x,y) = v^{'}(x,0) \cong U_\infty \left(\frac{dy}{dx}\right)_{\text{BODY}}$$

For three-dimensional planar flows

$$\frac{\partial f}{\partial z} \cong 0$$

and the boundary condition becomes

$$v^{'}(x,0,z) = U_\infty \left(\frac{\partial y}{\partial x}\right)_{\text{BODY}}$$

at infinity:

$$u^{'} \to 0$$
$$v^{'} \to 0$$
$$w^{'} \to 0$$

or $w^{'}$, $v^{'}$, and $w^{'}$ are finite.

7 Linearized Pressure Coefficient

Let's revisit the pressure coefficient, c_p:

$$c_p \equiv \frac{p - p_\infty}{\frac{1}{2}\rho_\infty U_\infty^2}$$

where p is the pressure (local, static) at the location or point of interest in the flow field. Note that c_p is dimensionless.
Since,

$$\frac{1}{2}\rho_\infty U_\infty^2 = \frac{1}{2}\frac{\gamma p_\infty}{\gamma p_\infty}\rho_\infty U_\infty^2 = \frac{\gamma}{2}p_\infty\frac{U_\infty^2}{a_\infty^2} = \frac{\gamma}{2}p_\infty M_\infty^2$$

then

$$c_p = \frac{2}{\gamma M_\infty^2}\left[\frac{p}{p_\infty} - 1\right]$$

for an inviscid, adiabatic, isentropic, steady flow and

$$\overline{Q} = (U_\infty + u')\overline{i} + v'\overline{j} + w'\overline{k}$$

We show that

$$h + \frac{1}{2}Q^2 = h_\infty + \frac{1}{2}U_\infty^2$$

which for a calorically perfect gas leads directly to

$$\begin{aligned}\frac{T}{T_\infty} &= \frac{\gamma-1}{2}\frac{U_\infty^2 - Q^2}{a_\infty^2}\\ &= 1 - \frac{\gamma-1}{2a_\infty^2}[2u'U_\infty + u'^2 + v'^2 + w'^2]\end{aligned}$$

Isentropic flow conditions lead to:

$$\frac{p}{p_\infty} = \left[\frac{T}{T_\infty}\right]^{\frac{\gamma}{\gamma-1}}$$

$$\frac{p}{p_\infty} = \left[1 - \frac{\gamma-1}{2}M_\infty^2\left(\frac{2u'}{U_\infty} + \frac{u'^2 + v'^2 + w'^2}{U_\infty^2}\right)\right]^{\frac{\gamma}{\gamma-1}}$$

In the case of small velocity perturbations,

$$\frac{u'}{U_\infty} \ll 1$$

$$\left(\frac{u'}{U_\infty}\right)^2 \lll 1$$

$$\left(\frac{v'}{U_\infty}\right)^2 \lll 1$$

$$\left(\frac{w'}{U_\infty}\right)^2 \lll 1$$

Using the binomial expansion, we show that

$$\frac{p}{p_\infty} = 1 - \frac{\gamma}{2}M_\infty^2\left(2\frac{u'}{U_\infty} + \frac{u'^2 + v'^2 + w'^2}{U_\infty^2}\right) + \dots$$

therefore:

$$c_p = -\frac{2u'}{U_\infty}$$

Discuss the limitations implied in the above expression for c_p.

8 Crocco's Theorem

Consider the motion of a fluid element. The fluid element may both translate and rotate.
Let:

$$\overline{v} = \text{translational velocity}$$

$$\overline{w} = \text{rotational velocity}$$

$$\overline{\omega} = \text{angular velocity}$$

where

$$\overline{w} = \frac{1}{2}\nabla \times \overline{v}$$

$$\nabla \times \overline{v} \equiv \text{vorticity}$$

Combine Euler's equation, first and second laws of thermodynamics:

$$\rho\frac{\partial \overline{v}}{\partial t} + \rho(\overline{v}\cdot\nabla)\overline{\nabla} = -\nabla p$$

$$T\nabla s = \nabla h - v\nabla p = \nabla h - \frac{\nabla p}{\rho}$$

$$h = h_0 - \frac{v^2}{2}$$

We obtain:

$$T\nabla s = \nabla h_0 - \overline{v} \times (\nabla \times \overline{v}) + \frac{\partial \overline{v}}{\partial t} \qquad \Big\{ \text{Crocco's Theorem}$$

For steady flow, we obtain

$$T\nabla s = \nabla h_0 - \overline{v} \times (\nabla \times \overline{v})$$

or

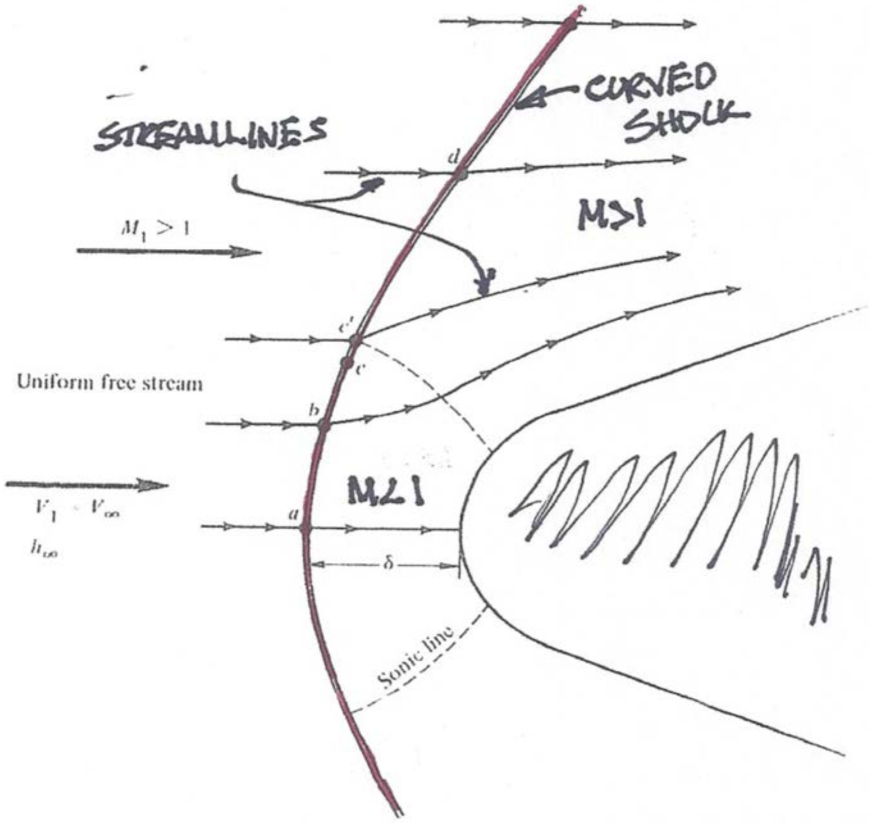
$$\overline{v} \times (\nabla \times \overline{v}) = \nabla h_0 - T\nabla s$$

For two-dimensional, steady flows:

$$2w = \frac{1}{v}\left(T\frac{\partial s}{\partial n} - \frac{\partial h_0}{\partial n}\right)$$

Vorticity $\implies$ rates of change of entropy and stagnation enthalpy normal to the streamlines

Flow over a supersonic blunt body:



For this flow,

$$h_0 = \text{constant}$$

$$\frac{\partial h_0}{h} = 0$$

$$\frac{\partial s}{\partial n} \neq 0 \qquad \text{(why?)}$$