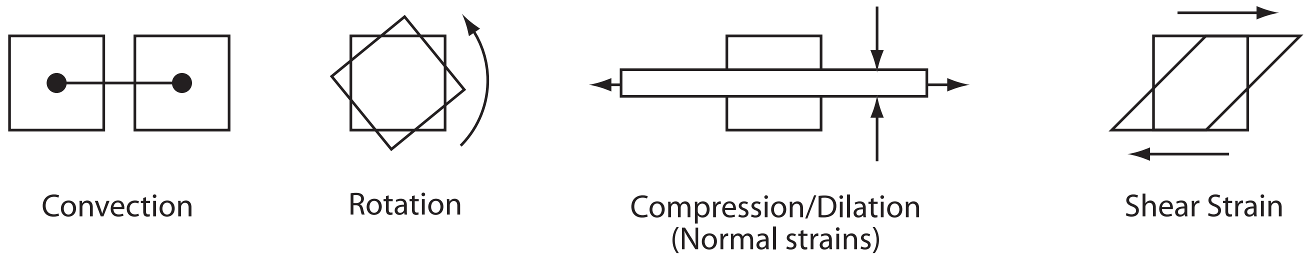
Kinematics of a Fluid Element



Convection: $\vec{u}$

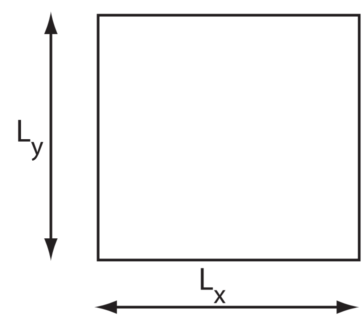
Rotation rate: $$\vec{\Omega} = \frac{1}{2}\nabla \times \vec{u} = \frac{1}{2}\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix}$$

$$\vec{\omega} = vorticity$$

$$= \frac{1}{2}\left\{\left[\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right]\vec{i} + \left[\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right]\vec{j} + \left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right]\vec{k}\right\}$$

Normal strain rates:

$$\varepsilon_{xx} = \frac{\frac{dL_x}{dt}}{L_x} = \frac{\partial u}{\partial x}$$
$$\varepsilon_{yy} = \frac{dL_y}{dt} = \frac{\partial v}{\partial z}$$
$$\varepsilon_{ZZ} = \frac{dL_z}{dt} = \frac{\partial w}{\partial z}$$



Shear strain rates:

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) = \frac{1}{2}\frac{d}{dt}\left(\begin{matrix}\text{Angle between edge} \\ \text{along } i \text{ and along } j\end{matrix}\right) = \varepsilon_{ji}$$

Strain rate tensor:

$$\begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{bmatrix}$$

Divergence

$$\nabla \bullet \vec{u} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = \frac{d\left(Volume\right)}{dt} / Volume$$

Substantial or Total Derivative

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \underbrace{u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}}_{\vec{u}\bullet\nabla}$$

=rate of change (derivative) as element move through space

Cylindrical Coordinates

$$\vec{u} = u_x\vec{e}_x + u_r\vec{e}_r + u_\theta\vec{e}_\theta$$

$$\varepsilon_{xx} = \frac{\partial u_x}{\partial x} \qquad \varepsilon_{rr} = \frac{\partial u_r}{\partial r} \qquad \varepsilon_{\theta\theta} = \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{u_r}{r}$$

$$\varepsilon_{r\theta} = \frac{1}{2}\left[r\frac{\partial}{\partial r}\left(\frac{u_\theta}{r}\right) + \frac{1}{r}\frac{\partial u_r}{\partial \theta}\right]$$

$$\varepsilon_{rx} = \frac{1}{2}\left[\frac{\partial u_r}{\partial x} + \frac{\partial u_x}{\partial r}\right]$$

$$\varepsilon_{\theta x} = \frac{1}{2}\left[\frac{1}{r}\frac{\partial u_x}{\partial \theta} + \frac{\partial u_\theta}{\partial x}\right]$$

$$\nabla \times \vec{u} = \left[\frac{1}{r}\frac{\partial}{\partial r}\left(ru_\theta\right) - \frac{1}{r}\frac{\partial u_r}{\partial \theta}\right]\vec{e}_x + \left[\frac{1}{r}\frac{\partial u_x}{\partial \theta} - \frac{\partial u_\theta}{\partial x}\right]\vec{e}_r + \left[\frac{\partial u_r}{\partial x} - \frac{\partial u_x}{\partial r}\right]\vec{e}_\theta$$

$$\nabla \bullet \vec{u} = \frac{\partial u_x}{\partial x} + \frac{1}{r}\frac{\partial\left(ru_r\right)}{\partial r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta}$$