16.06 Principles of Automatic Control
Lecture 19

Bode Plots With Complex Poles

Second-order term: Usually in denominator:

$$\frac{1}{(\frac{s}{\omega_n})^2 + 2\zeta\frac{s}{\omega_n} + 1}$$

This is plotted much like first-order term, except slope in high-frequency regime is -2 (-40 dB/dec).

$$|G(j\omega_n)| = \frac{1}{2\zeta}$$
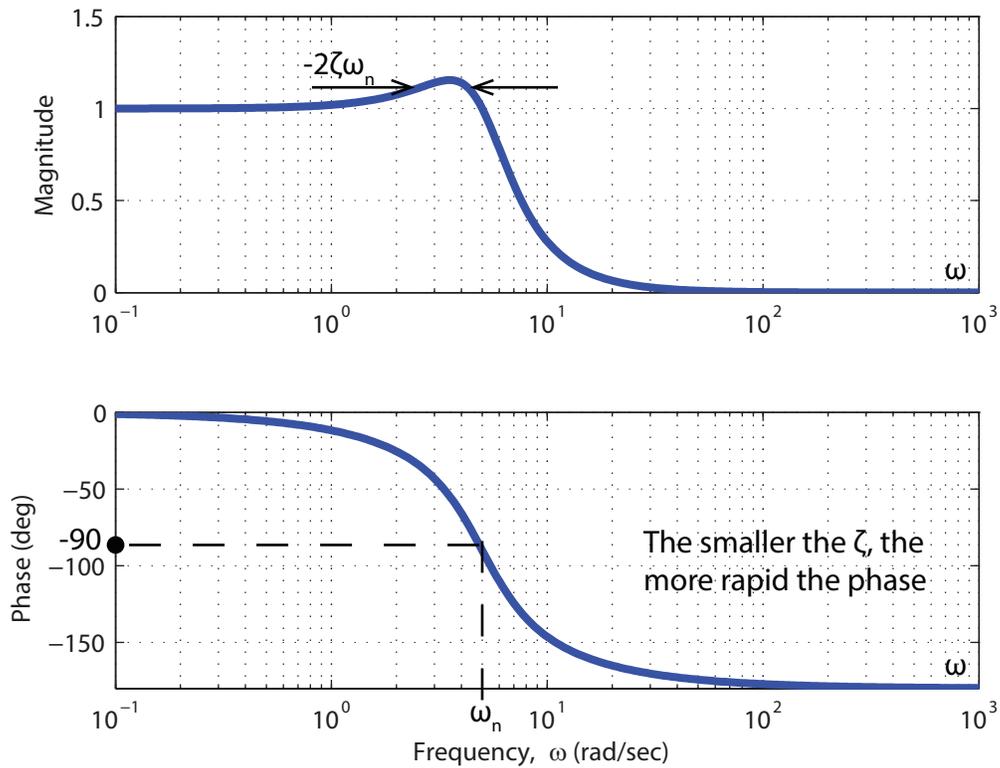
How rapid is phase change?

Im(s)

phase changes by about ±45°,
as ω goes from ω(1-ζ) to ω(1+ζ).

Re(s)
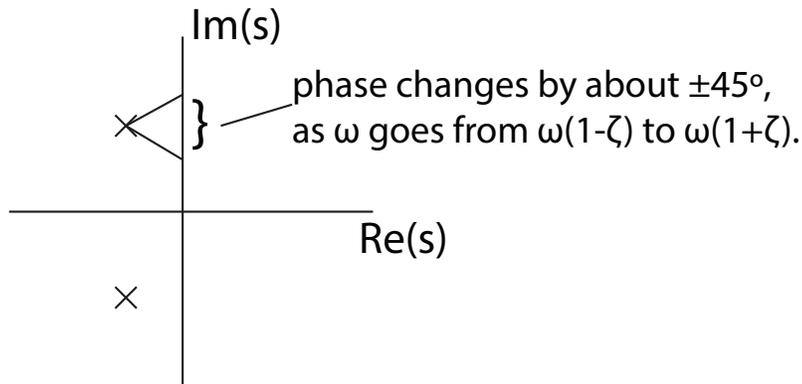
Magnitude changes from $\frac{1}{2\sqrt{2}\zeta}$ to $\frac{1}{2\zeta}$ to $\frac{1}{2\sqrt{2}\zeta}$ in same range. So width of peak/phase change is about $2\zeta\omega_n$.

Nonminimum Phase Systems

A system with a zero in the RHP is called a *nonminimum phase system.* Consider

$$G_1(s) = \frac{1 + s/10}{1 + s/1}$$

and

$$G_2(s) = \frac{1 - s/10}{1 + s/1}$$

G_2 has a zero at $s = +10$, and so is NMP.

Let's look at the magnitude and phase of each transfer function:

$$M_1 = |G_1(j\omega)| = \left| \frac{1 + j\omega/10}{1 + j\omega/1} \right|$$

$$= \left(\frac{1 + \omega^2/10^2}{1 + \omega^2/1^2} \right)^{1/2}$$

$$\phi_1 = \angle G_1(j\omega) = \angle \frac{1 + j\omega/10}{1 + j\omega/1}$$

$$= \tan^{-1}\omega/10 - \tan^{-1}\omega/1$$

The magnitude of G_2 is similar:

$$M_2 = |G_2(j\omega)| = \left| \frac{1 - j\omega/10}{1 + j\omega/1} \right|$$

$$= \left(\frac{1 + \omega^2/10^2}{1 + \omega^2/1^2} \right)^{1/2}$$

$$= M_1$$

So G_1 and G_2 have the same magnitude plots.

The phase of G_2 is:

$$\phi_2 = \angle G_2(j\omega) = \angle \frac{1 - j\omega/10}{1 + j\omega/1}$$

$$= \tan^{-1}(-\omega/10) - \tan^{-1}(\omega/1)$$

$$= - \underbrace{\tan^{-1}\omega/10}_{\substack{\text{same term as in } \angle G_1.. \\ \text{but with a minus sign}}} - \underbrace{\tan^{-1}\omega/1}_{\text{same as term in } \angle G_1}$$

So G_1 and G_2 have the same magnitude, but G_2 always has more negative phase.

Note that we can write

$$G_1(s) = G_2(s) \underbrace{\frac{1 - s/10}{1 + s/10}}$$

> This part has magnitude 1, and phase $-2\tan^{-1}\omega/10$, which is negative for all $\omega > 0$. This result is general - for any system G_1 with RHP zeros, G_1 can be expressed as

$$G_1(s) = \underbrace{G_2(s)}_{\text{has only LHP zeros}} \cdot \underbrace{U(s}_{M=1,\ \phi<0}$$

A system with poles and zeros in LHP is called minimum phase, because it has less phase *lag* than any stable system with the same magnitude plot. Therefore, a system with RHP zeros is called *non*minimum phase.

Two reasons to worry about NMP systems:

1. Usual Bode Rules for phase are backwards, so it's easy to make mistakes.

2. Phase lag is bad, and NMP systems have "excess" phase lag! As we will see, this places limitations on the performance we can achieve using feedback control.

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