16.06 Principles of Automatic Control
Lecture 18

Bode Plot Construction (continued)

Note that phase of s^α term is

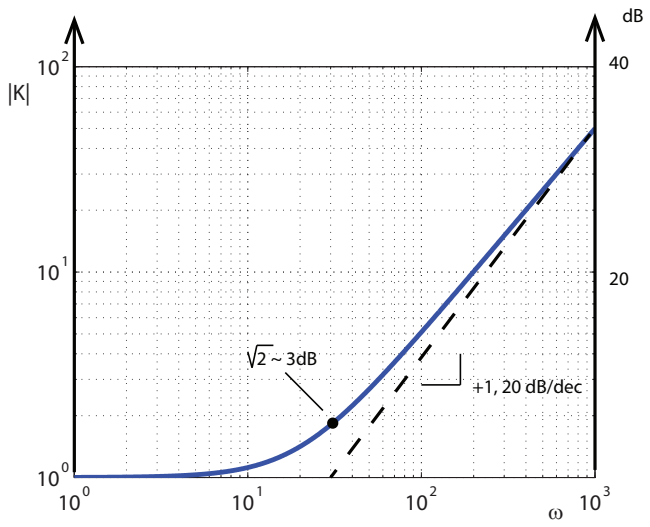
$$\begin{aligned}\angle(j\omega)^\alpha =& \angle j^\alpha = \alpha \angle j \\ =& \alpha \cdot 90^\circ\end{aligned}$$

To plot $1 + s/a$ term, note that

$$\begin{aligned}|1 + j\omega/a| =& (1 + \omega^2/a^2)^{1/2} \\ =& \begin{cases} 1, & \omega \ll a \\ \omega/a, & \omega \gg a \\ \sqrt{2}, & \omega = a \end{cases}\end{aligned}$$

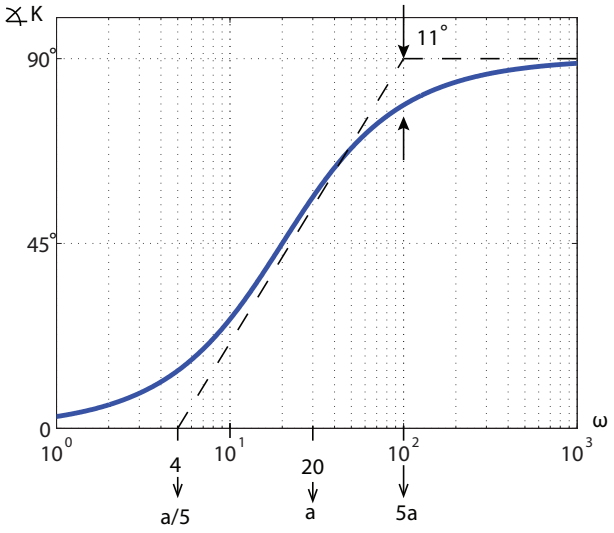
Example:

$K(s) = 1 + s/20$



What about the phase?

$$\angle 1 + j\omega/a = \tan^{-1}\omega/a$$

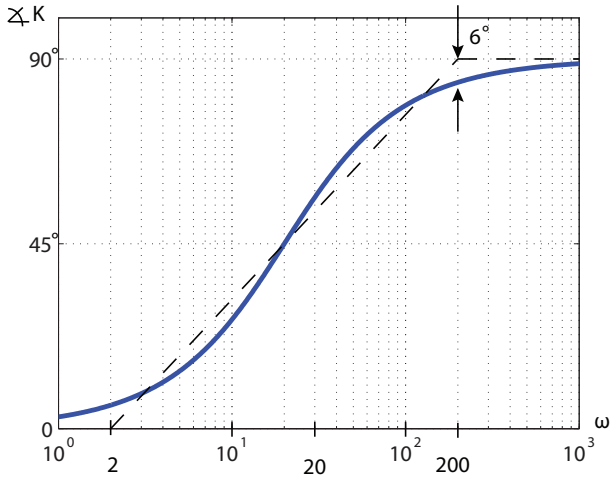


That is, the phase varies by 90°over the frequency range $(\frac{a}{5}, 5a)$.

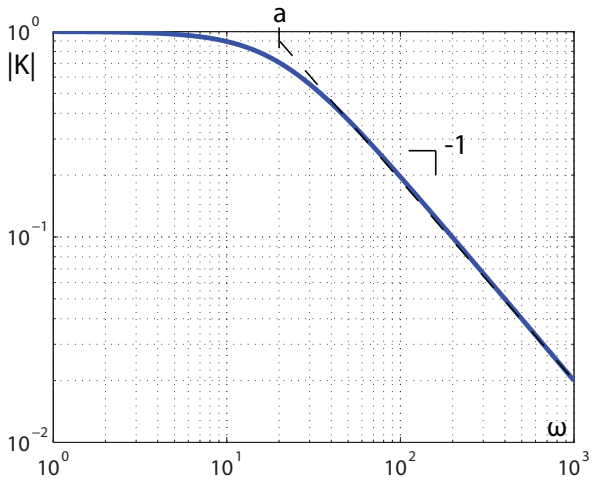
Some people find it easier to draw the construction lines with breakpoints at $a/10, 10a$.

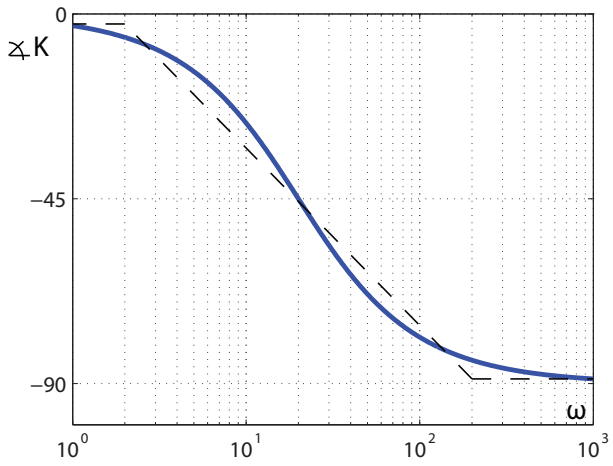
- Easier to draw
- Less phase error

- Middle segment is not technically an asymptote anyway



For $K = \frac{1}{1+s/a}$, the above magnitude and phase plots are flipped about $|K| = 1$ or $\angle K = 0°$.





Bode Rules:

Rule 1: Manipulate the transfer function into Bode form.

Rule 2: Determine α for $K_0 s^\alpha$ term. Plot the low-frequency asymptote with slope α(or 20α dB/dec) through the point $\omega = 1, \quad 1 \cdot 1 = K_0$.

Rule 3: Complete the composite magnitude asymptotes. At each break point, change the slope by ± 1, or ± 2, as appropriate.

Rule 4: Sketch in approximate magnitude curve. (see FPE for more details).

Rule 5: Plot the low frequency asymptote of the phase curve ($\phi = \alpha \cdot 90^\circ$).

Rule 6: The approximate phase is found by changing the phase by $\pm 90^\circ$or $\pm 180^\circ$ at each breakpoint.

Rule 7: Locate the asymptotes for each phase curve, at break points 1/5 and 5 times (or 1/10 and 10 times) the frequency of the magnitude break point.

Rule 8: Graphically add the asymptotes, and draw the approximate phase curve.

Example:

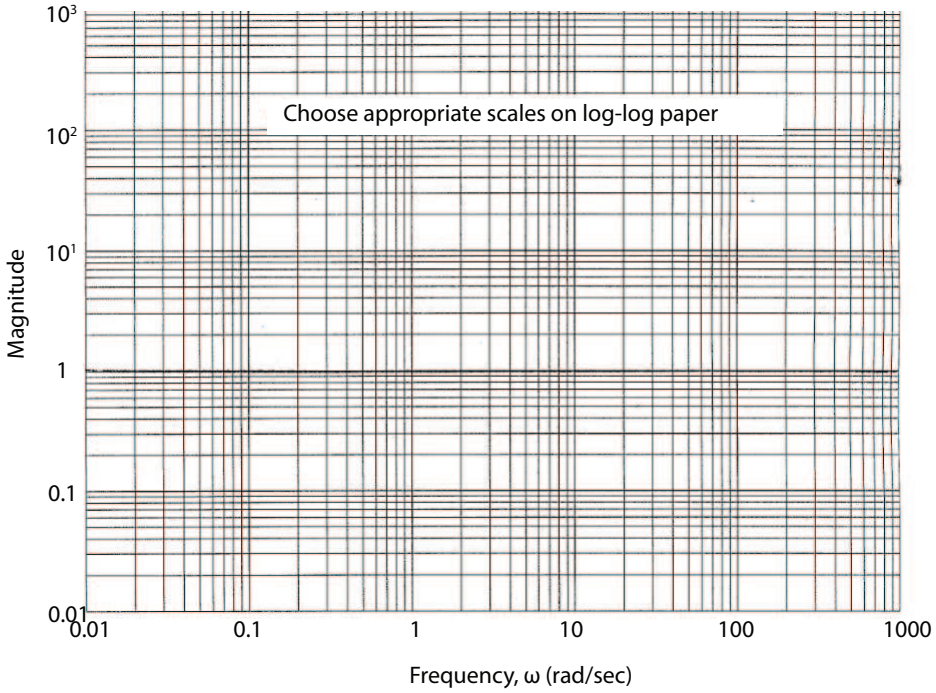
$$\begin{aligned} KG(s) =& \frac{2000(s+0.5)}{s(s+10)(s+50)} \\ =& \frac{2(1+s/0.5)}{s(1+s/10)(1+s/50)} \end{aligned}$$

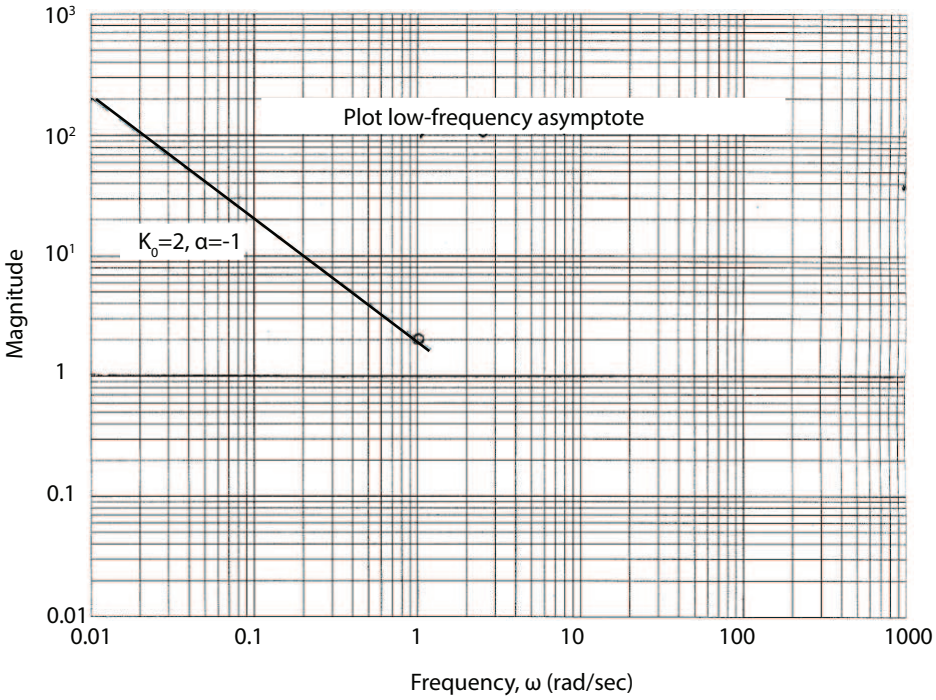
The magnitude break points are

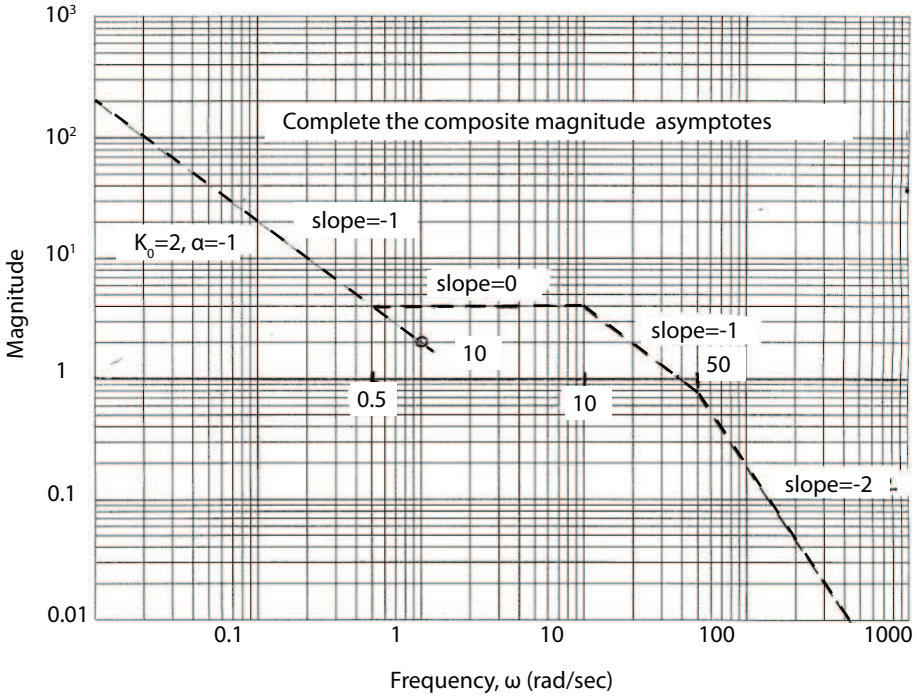
$$\begin{array}{ll} 0 & 0.5 \\ x & 10 \\ x & 50 \end{array}$$

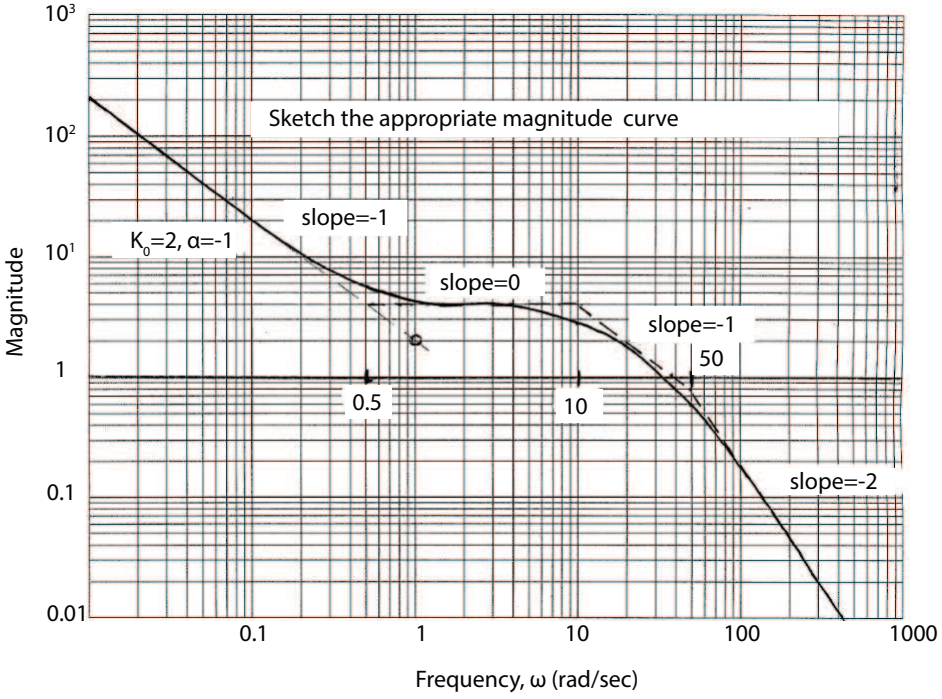
The phase break points are

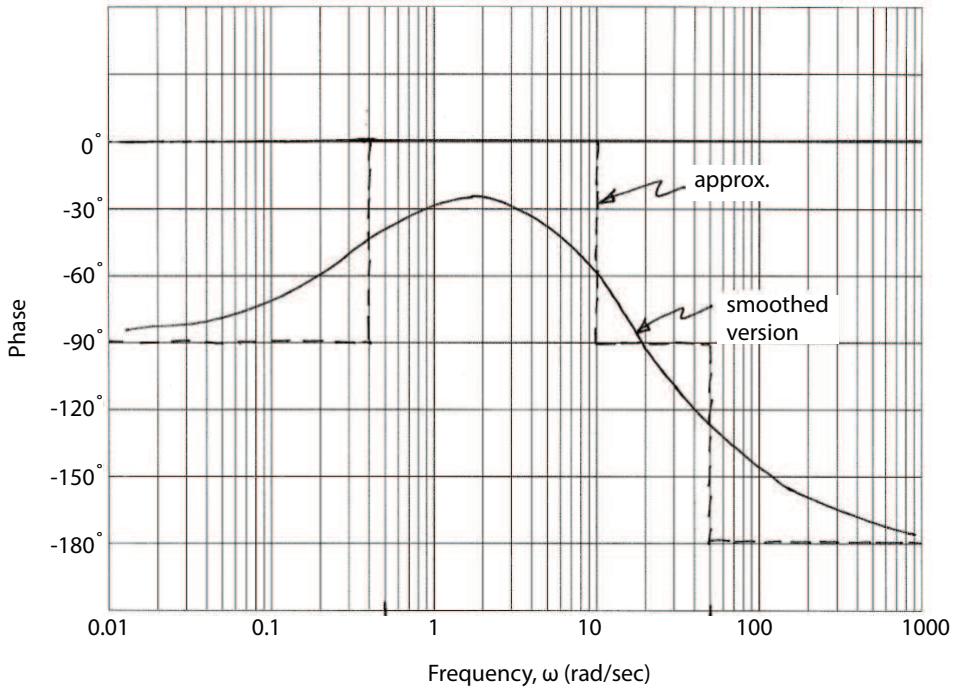
$$\begin{array}{ll} 0 & 0.05, 5 \\ x & 1,100 \\ x & 5,500 \end{array}$$

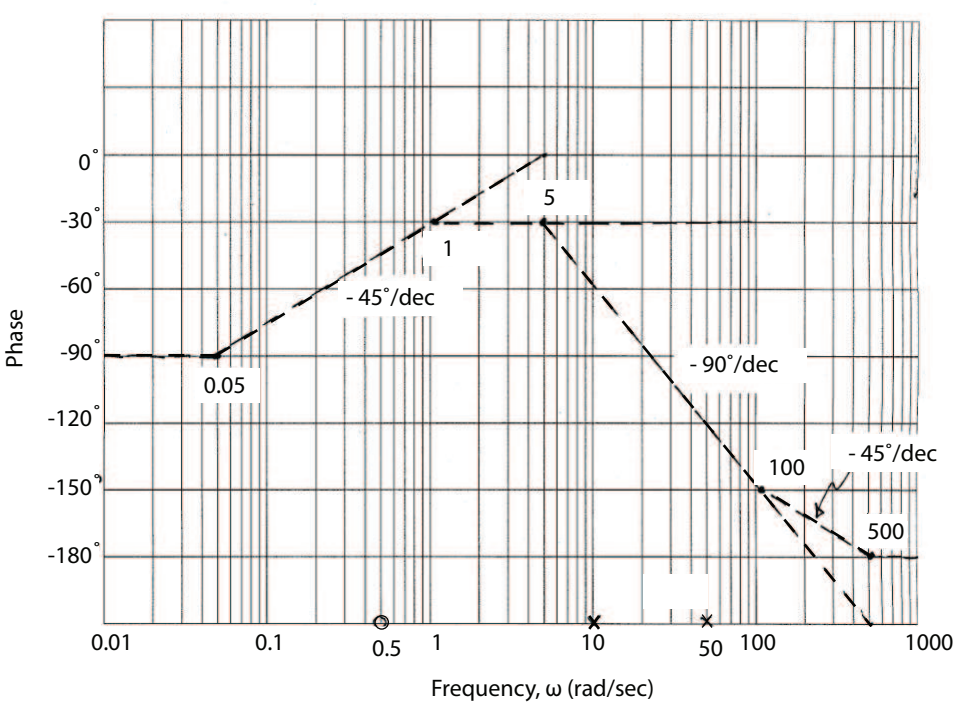

Choose appropriate scales on log-log paper
Magnitude
10^3
10^2
10^1
1
0.1
0.01
0.01
0.1
1
10
100
1000
Frequency, ω (rad/sec)

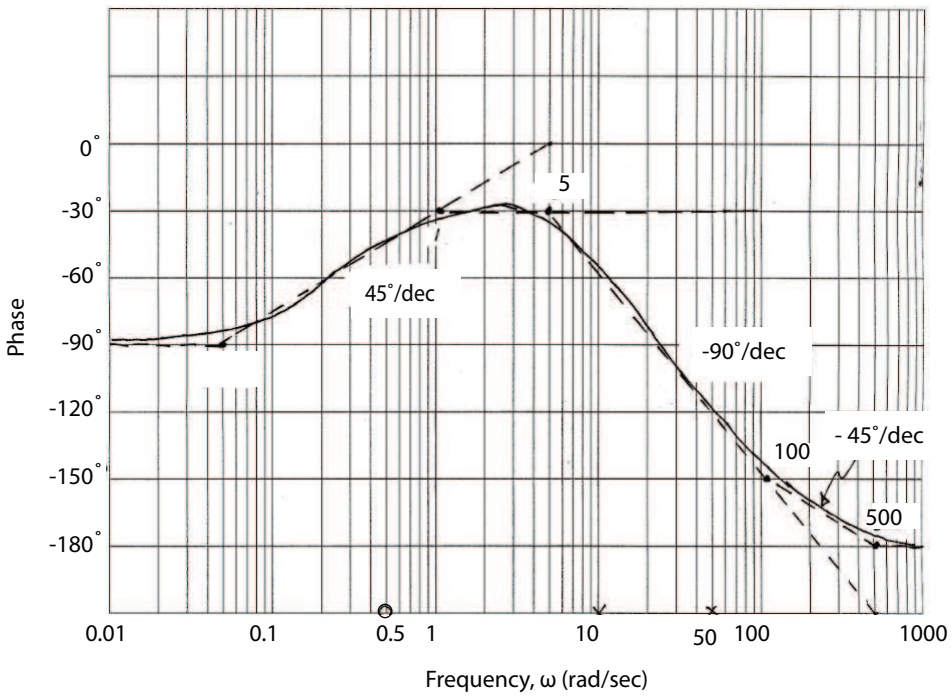

Plot low-frequency asymptote
K_0=2, α=-1
Magnitude
10^3
10^2
10^1
1
0.1
0.01
0.01
0.1
1
10
100
1000
Frequency, ω (rad/sec)


Complete the composite magnitude asymptotes
slope=-1
K_0=2, α=-1
slope=0
slope=-1
slope=-2
10
50
0.5
10
Magnitude
Frequency, ω (rad/sec)


Sketch the appropriate magnitude curve
slope=-1
K_0=2, α=-1
slope=0
slope=-1
slope=-2
50
0.5
10
Magnitude
Frequency, ω (rad/sec)


0°
-30°
-60°
-90°
-120°
-150°
-180°
Phase
approx.
smoothed
version
0.01
0.1
1
10
100
1000
Frequency, ω (rad/sec)


0°
-30°
-60°
-90°
-120°
-150°
-180°
Phase
5
1
- 45°/dec
0.05
- 90°/dec
100
- 45°/dec
500
0.01
0.1
0.5
1
10
50
100
1000
Frequency, ω (rad/sec)


0°
-30°
-60°
-90°
-120°
-150°
-180°
Phase
5
45°/dec
-90°/dec
100
- 45°/dec
500
0.01
0.1
0.5
1
10
50
100
1000
Frequency, ω (rad/sec)

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