Improved Performance Estimates for Optimization 23 Mar 06

Lab 7 Lecture Notes

Nomenclature

ρ	air density	g	gravity
W	aircraft weight	P_{elec}	electric power
S	reference area (wing area)	$P_{\max}$	maximum thrust power ($= (\eta P_{\mathrm{elec}})_{\max}$)
b	wing span	η	overall propulsive efficiency
c	average wing chord	C_L	lift coefficient
λ	taper ratio	c_d	wing profile drag coefficient
AR	wing aspect ratio	CDA_0	drag area of non-wing components
I_o	wing root bending inertia	τ	airfoil thickness/chord ratio
E	Young's modulus	δ	tip deflection

Assumed Design Variables

The primary design variables here are assumed to be AR and S. Other design variables such as τ, λ, etc., may also be considered, although for brevity these will not be shown in the argument lists below.

The following dependent auxilliary functions and constraint functions are assumed to be known from suitable analyses:

$$c(AR, S) = \sqrt{S/AR} \tag{1}$$

$$b(AR, S) = \sqrt{S \times AR} \tag{2}$$

$$W_{\mathrm{wing}}(AR, S) = 0.6\,\tau\,c^2\,b\,\rho_{\mathrm{foam}}\,g \tag{3}$$

$$W(AR, S) = W_{\mathrm{wing}} + W_{\mathrm{fuse}} \tag{4}$$

$$I_0(AR, S) = 0.04\,\tau^3\,c^4\left(\frac{2}{1+\lambda}\right)^4 \tag{5}$$

$$\frac{\delta}{b}(AR, S) = \frac{W_{\mathrm{fuse}}}{EI_0}\,\frac{b^2}{96}\,\frac{1+2\lambda}{1+\lambda} \tag{6}$$

The velocity for level flight and corresponding chord Reynolds number are also convenient auxilliary functions.

$$V(AR, S) = \left(\frac{2W}{\rho\,S\,C_L}\right)^{1/2} \tag{7}$$

$$Re(AR, S) = \frac{V\,c}{\nu} \tag{8}$$

Minimum Flight Power

The battery power needed to sustain level flight is given by the flight power relation.

$$P_{\mathrm{elec}}(AR, S) = \frac{1}{\eta}\,\frac{1}{2}\rho V^3\,S\left[\frac{CDA_0}{S} + c_d + \frac{C_L^2}{\pi\,e\,AR}\right] \tag{9}$$

Most of the variables in this expression depend implicitly on the design variables. The $W(AR, S)$ and $V(AR, S)$ functions were considered earlier. The overall power conversion efficiency depends on the velocity,

$$\eta(V)$$

which is evaluated in terms of the design variables by direct substitution of expression (7).

$$\eta(AR, S) \;=\; \eta(V_{(AR,S)}) \tag{10}$$

Similarly, the profile drag depends on the C_L and Reynolds number,

$$c_d(C_L, Re)$$

which again can likewise be given in terms of the design variables.

$$c_d(AR, S) \;=\; c_d(C_L, Re_{(AR,S)}) \tag{11}$$

Maximum Flight Speed

The maximum flight speed $V_{\max}$ is given implicitly by the flight power relation (9), with P_{elec} set to the maximum available power. It is convenient to consider P_{elec} and η together, as the maximum available thrust power $P_{\max}(V) = (\eta P_{\text{elec}})_{\max}$.

$$P_{\max} \;=\; \frac{1}{2}\rho V_{\max}^3 S \left[\frac{CDA_0}{S} \;+\; c_d(C_{L_{\min}}, Re_{\max}) \;+\; \frac{C_{L_{\min}}^2}{\pi\, e\, AR} \right] \tag{12}$$

$$C_{L_{\min}} \;=\; \frac{2W/S}{\rho V_{\max}^2} \tag{13}$$

$$Re_{\max} \;=\; \frac{V_{\max}\, c}{\nu} \tag{14}$$

Although equations (12), (13), (14) cannot be explicitly solved for $V_{\max}(AR, S)$, it is possible to solve them by a reasonably simple iterative procedure. We note that at maximum speed the induced drag is likely to be small, and the profile c_d only weakly dependent on V. Hence, the initial $V_{\max}$ value can be guessed, and then subsequently improved. The iteration proceeds as follows:

0) Assume some expected $\tilde{V}_{\max}$ value.

1) Compute corresponding $\tilde{C}_{L_{\min}}$.

$$\tilde{C}_{L_{\min}} \;=\; \frac{2W}{\rho \tilde{V}_{\max}^2 S} \tag{15}$$

2) Evaluate all other quantities which depend on V or C_L.

$$\tilde{Re}_{\max} \;=\; \frac{\tilde{V}_{\max}\, c}{\nu} \tag{16}$$

$$\tilde{P}_{\max} \;=\; P_{\max}(V_{\max}) \tag{17}$$

$$\tilde{c}_d \;=\; c_d(\tilde{C}_{L_{\min}}, \tilde{Re}_{\max}) \tag{18}$$

$$\tilde{C}_{D_i} \;=\; \frac{C_{L_{\min}}^2}{\pi e AR} \tag{19}$$

3) Calculate improved $V_{\max}$ from power relation (12).

$$V_{\max} \;=\; \left[\frac{2\,\tilde{P}_{\max}}{\rho\,(CDA_0 + S\tilde{c}_d + S\tilde{C}_{D_i})} \right]^{1/3} \tag{20}$$

The iteration can be repeated by starting again at 1). Only one such additional pass is likely to be necessary in most cases.