

## Lecture S11 Muddiest Points

### General Comments

In this lecture, we looked at the definition of *state*, and applied state-space ideas to the derivation of the dynamics of a circuit. This approach leads to a set of first order, linear differential equations in a specific form. Hopefully, it will become a little more clear in the next lecture why this form is so nice.

### Responses to Muddiest-Part-of-the-Lecture Cards

(36 cards)

1. *What are state vectors used for? How does the equation*

$$\dot{\underline{x}} = A\underline{x} \tag{15}$$

*predict the behavior of the circuit? (1 student) I don't really understand the point behind state functions and how to derive them. (1)* The state equation (in matrix / vector form) is a set of first order, coupled differential equations that describe the dynamics of the circuit, just as were the equations we found in previous lectures. The difference is that the state-space form is a special case, that expresses the dynamics in a very specific form that is easier to work with. It also has theoretical advantages that are difficult to explain in only one or two lectures, but is truly the modern way to think about the dynamics of systems. For example, the state-space approach made possible advances in navigation technology that enabled travel to the moon.

2. *In the example, why does  $e_1 = v_4$ ? (1)* The inductor  $L_4$  is connected to the upper node with potential  $e_1$ , and the ground node. The difference in potential across the inductor is then  $e_1 - 0 = e_1$ .
3. *Confused about the derivation of the state equations. (1) General confusion about the states of the circuit.* Please re-read the notes, and see me at office hours Monday, or in recitation.
4. *Your example was clear up to the application of KCL at  $v_1$ . (2)* After solving for all the unknown nodes, we solve for the current in the voltage sources (or capacitor in this case). The current out of the  $v_1$  node is

$$i_1 + G_2 v_1 + G_3 (v_1 - e_1) \tag{16}$$

so we can conclude that

$$i_1 = -G_2 v_1 - G_3 (v_1 - e_1) \tag{17}$$

But we already found that

$$e_1 = v_1 - R_3 i_4 \tag{18}$$

Therefore

$$i_1 = -G_2 v_1 - i_4 \tag{19}$$

5. *How did*

$$R_3 i_4 + e_1 - v_1 = 1 \quad (20)$$

*become*

$$v_4 = e_1 = c_1 - R_3 i_4 \quad (21)$$

*(Card shows big dot over the  $i_4$ .) Where did the dot come from? (1)* It doesn't have a differentiation dot over it — maybe that wasn't clear from my blackboard technique.

6. *Please explain how to find the number of states. (1) I don't understand how to pick the state variables.* The most straightforward to determine the state variables is to find the initial conditions required to simulate or predict the future behavior of the system.
7. *In the example worked in class, we know that  $v_2 = v_1$  (because  $C_1$  and  $R_2$  are in parallel), and we know  $i_4$ . Isn't it true that we can immediately solve for  $i_1 = -i_2 - i_4$ ? (1)* Yes, but in more complicated circuits, it would not usually be so obvious. I'm trying to do the examples the way I would do larger problems.
8. *In the PRS question, you said you would have said that there are 9 states, but we only had 7 on the board. What are the others? (1)* The nine states are the 3 components of position, the 3 components of velocity, and 3 components of angular rate.
9. *What are the variables with dots? (1)* Time derivatives. For example,  $\dot{v}(t)$  is the same as  $dv(t)/dt$ .
10. *The state-space approach doesn't appear to be less messy than the previous method. (1)* The derivation is not less messy, you're right. However, there is a rich, complete theory of how to find solutions for state-space equations. It's harder to develop a general theory for non-state space systems. All of "modern control theory," for example, is based on state-space descriptions of systems.
11. *When do we use imaginary numbers in the generalized impedance equation you gave us? In 8.022, we used  $Z = i/Cs$ , or  $Z = iLs$ . (1)* Are you sure it wasn't  $Z = iL\omega$ ? In general,  $s$  can be complex. For sinusoidal input,  $s = i\omega$ , so  $Ls = iL\omega$ .
12. *How can you replace a capacitor with a voltage source, and an inductor with a current source, and keep the accuracy? (1)* The circuit with sources replacing the capacitors and inductors is an analysis circuit, that is used to determine the current flow through the capacitor, and the voltage across the inductor. We're not saying that they are the same circuit; we're saying that the resulting currents and voltages are the same.
13. *Can you restate the definition of linearity with respect to circuits, and give an example of nonlinear elements? (1)* Circuit elements are linear if their constitutive relationship is a linear relationship between  $i$  and  $v$ . For a resistor,  $v = iR$ , and  $v$  is a linear function of  $i$ , so a resistor is linear. For a capacitor,  $i = C dv/dt$ . In this case,  $C d/dt$  is a linear operator on  $v$ , so a capacitor is linear. We will give a broader definition of linearity in the spring term.

An example of a nonlinear element is a diode. An ideal diode allows any current flow in one direction (with no voltage drop), but no current flow in the other direction, no matter the applied voltage.

14. ***In the PRS question, would wind be a variable? (1)*** Probably not, for two reasons: (1) There isn't much wind in a basketball arena. (2) Usually, wind would be considered to be a parameter (if the wind is constant), or an exogenous input (if it varies in an unpredictable way). However, if you set out to model the wind variation, there could be one or more states associated with the wind. Aerodynamicists often use a wind turbulence model known as the Dryden model, which has 6 states, two for each direction in space.
15. ***What are the three ways to identify states? (1)*** The states are always associated with *initial conditions* that must be given to predict the future behavior of the system. The usually arise from variables that have *time derivatives* in the differential equations. These are very often (but not always) associated with energy storage. For example, velocity is usually a state in systems with kinetic energy.
16. ***Is the state vector just a collection of data or does it have direction as well? (1)*** Usually, the state vector is a mathematical vector, not a physical or geometric vector. As such, it is more a collection of data, and does not have a "direction" that you can physically point to. However, geometric ideas are useful in gaining understanding about state spaces, so it is common for folks to talk about the "direction" in state space.
17. ***Why are states associated with energy sources? (1)*** States are often associated with energy storage. For example, the voltage across a capacitor is often a state variable, because there is energy storage ( $Cv^2/2$ ). Such elements are associated with states, because to change the energy, power must be applied to the element (by conservation of energy). As a result, there is always a differential equations associated with an energy storage element, and therefore there must be an initial condition to keep track of.
18. ***In the definition of state, what did you mean by "variables of smallest size"? (1)*** I said "a set of variables of smallest size." The set is as small as possible (fewest number of variables). It's not the variables that are small.
19. ***What is dispersion? (2)*** In a medium such as air, sound waves travel at a fixed speed, regardless of the frequency of vibration. Such a medium is *nondispersive*. In a beam, sound waves travel at different speeds, depending on the frequency. So high-frequency sound travels faster than low-frequency sound. When waves travel at different speeds depending on frequency, the medium is *dispersive*.
20. ***No mud. (12)***