Statically Indeterminate Structures and Calculation of Deflections

Deflection of Trusses

Need other two great principles

- constitutive behavior
- compatibility

Constitutive Behavior for Bars in a Truss

Will later show (blocks 2 & 3) that the overall change in length of bar is (for some range of materials, loads)

$$\delta = \underbrace{\frac{FL}{AE}}_{\boxed{\text{Due to force}}} + \underbrace{\alpha \Delta T L}_{\boxed{\text{Due to a change in temperature}}}$$

Symbol	Meaning	Dimensions	SI units
$A =$	Cross-section area	$\left[L^2\right]$	m^2
$L =$	Length of bar	$[L]$	m
$F =$	Applied force	$[ML/T^2]$	$N/$
$E =$	Modulus of elasticity	$[ML/T^2.L^2]$	N/M^2
$\alpha =$	Coefficient of thermal expansion	$\left[{}^{L}/_{L.\theta}\right]$	${}^{M}/_{M}K$
$\Delta T =$	Temperature difference	$[\theta]$	K

Check Units

$$\delta = \frac{\left[{ML}/{T^2} \right][L]}{\left[L^2\right]\left[{ML}/{T^2 L^2} \right]} + \left[\frac{L}{L\theta}\right][\theta][L]$$

$$= [L] + [L] \rightarrow \quad OK$$

This is basically the same as $F = K\delta$ for zero thermal expansion

Set $\quad \Delta T = 0 \quad then \quad \delta = \dfrac{FL}{AE}$

$$rearranging \quad \underbrace{\left(\frac{AE}{L}\right)}_{\boxed{k \text{ for a solid bar}}}\delta = F$$

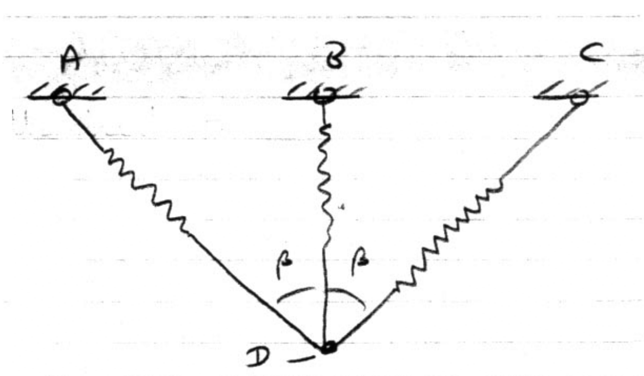
[NOTE: a real spring has added geometry]

<u>Compatibility of Displacements</u>

"Configurations which are attached must have internal deformations consistent with the external displacements"

<u>Example</u>

3 springs
stiffness k



Springs stretch under loading, but remain attached at D.

$\therefore$ Deformations of AD, BD & CD must be compatible.

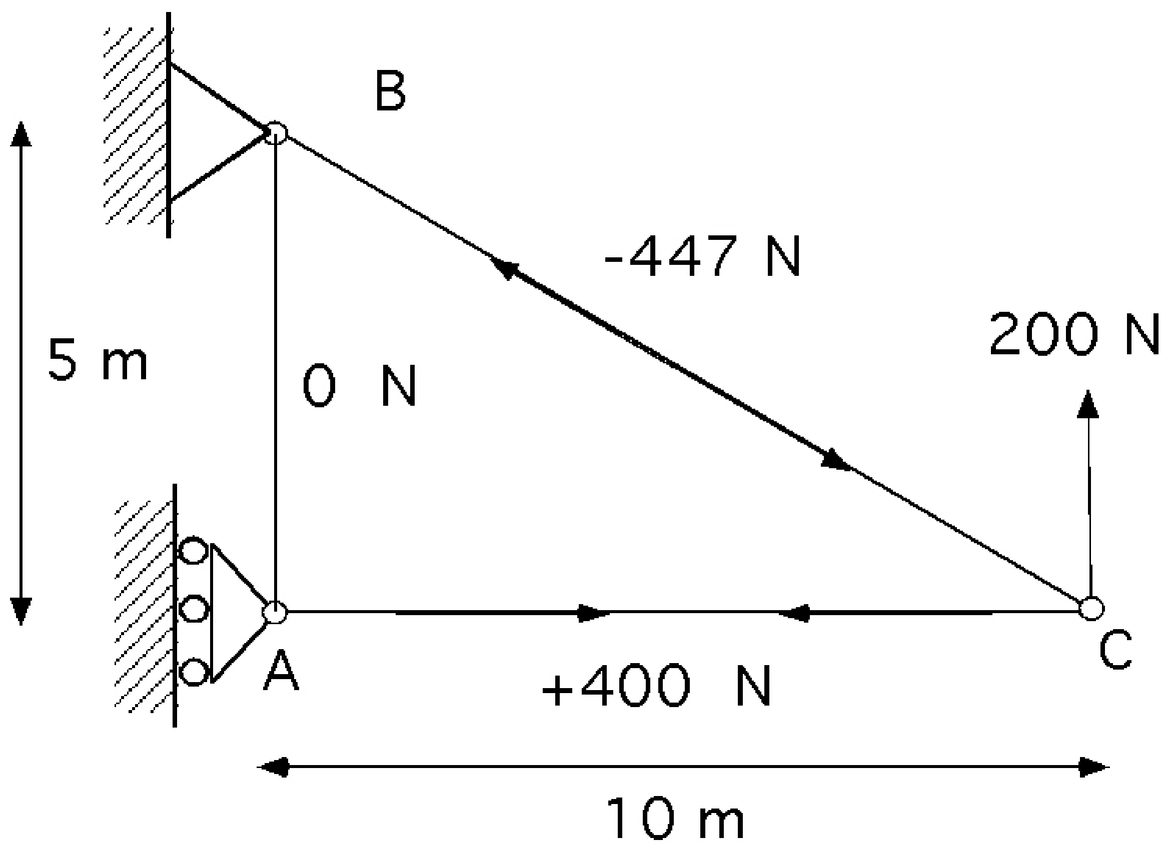
Compatibility For Trusses

For truss like structures:

- bars can extend/contract axially
- can rotate about pin joints
- but remain attached at joints

i.e., deflections and rotations must be compatible

Consider 3 bar truss from lecture M5, with deflections due to applied loading, no temperature change:



Tabulate bar forces and resulting extensions

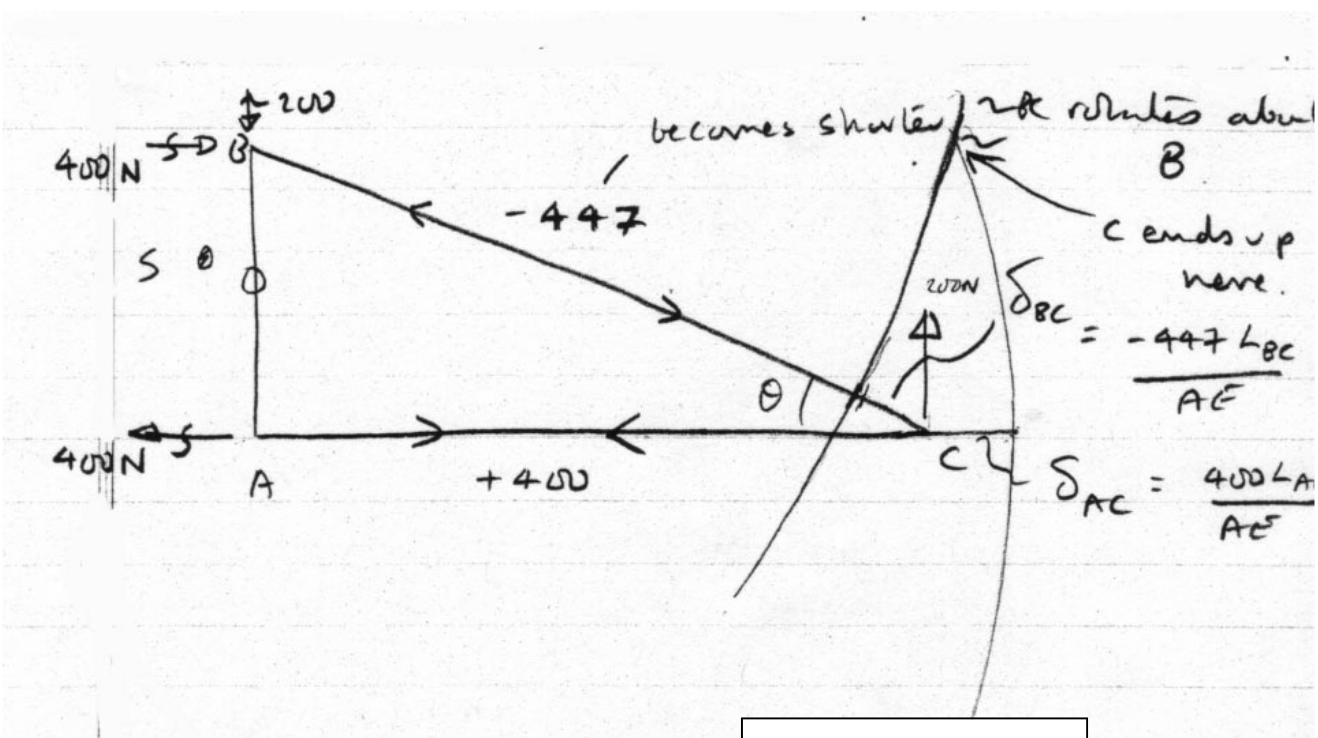
	Force/N	Length/m	$\frac{FL}{AE}$ (μm)
AB	*0*	*5*	*0*
AC	*+400*	*10*	*571*
BC	*-447*	$\sqrt{125}$	*-714*

$A = 1 \times 10^{-4} m^2$

$E = 70 GPa$

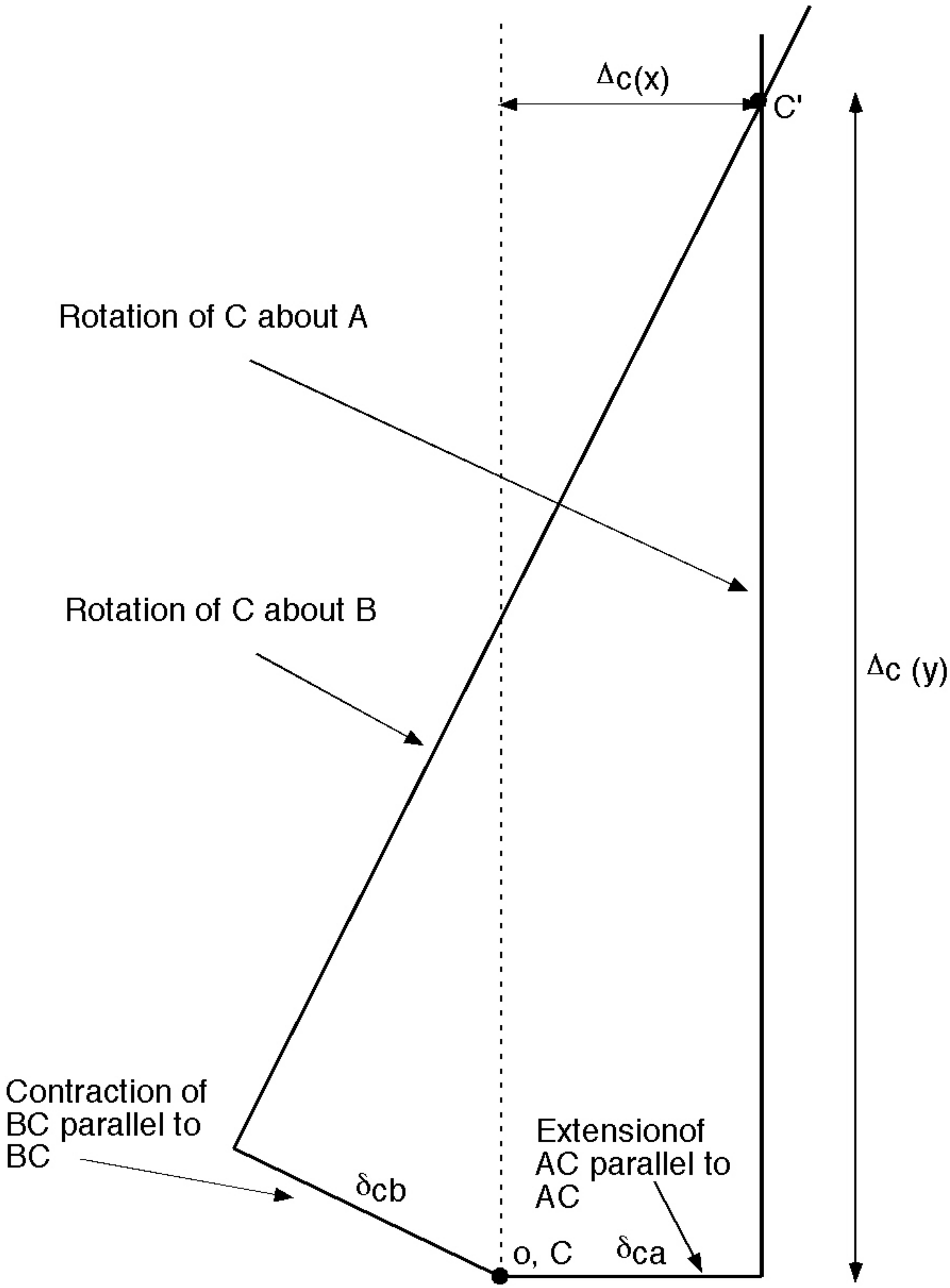
$AE = 7 \times 10^6 N$

Consider what this implies about deformations of 3 bar truss. Each bar extends or contracts, but they must remain connected at the joints. The bars must rotate about the joints to allow them to remain connected.



AC rotates about A

can enlarge deflections and rotations around in location of point C, assume deflections are small, draw a <u>displacement diagram.</u>



Can extend to other joints by considering relative displacements.

Displacement diagrams are effectively plotting the displacement vectors of the joints as defined by the end of the bars. The displacement vector for the end of a bar is made up of two components: (1) an extension, of a magnitude defined by the bar force and the constitutive behavior of the bar which is parallel to the direction of the bar and (2) a rotation, which is undefined in magnitude, but is perpendicular to the direction of the bar (on the displacement diagram).

TIP: Do this on graph paper - measure deflections rather than calculate. Or use a drawing program (Canvas™, Illustrator™ etc.)