Muddy Card Responses Lecture M12 2/22/2004

Synopsis. Introduced model for Euler buckling of a simply supported column. Key step is to allow the column to adopt a deformed (buckled) shape and to perform method of sections to obtain governing differential equation: $EI\dfrac{d^2w}{dx^2} + Pw = 0$. General solution of form:

$w = A\sin\sqrt{\dfrac{P}{EI}}x + B\cos\sqrt{\dfrac{P}{EI}}x + Cx + D$ where A, B, C, D are determined by boundary

conditions. For simple support conditions, solutions either A, B, C and D = 0 (trivial) or

$\sin\sqrt{\dfrac{P}{EI}}L = 0 \Rightarrow \sqrt{\dfrac{P}{EI}}L = n\pi$ leading to the condition on the load for buckling: $P = \dfrac{n^2\pi^2 EI}{L^2}$

- where n=1 the Euler buckling load.

I understand the math behind buckling, but I don't think that I fully understand the physics. If we are only applying an axial load, how do the internal forces in the beam push curvature into it? This is a really good question. If the beam were in fact perfect it could remain straight (unbuckled) under an axial load P above the buckling load. In practice there will always be a slight imperfection (non-axial loading or an initial curvature) that will result in the beam being pushed into the buckled curvature. We can discuss in M13.

In the last question, Ben asked why is A=0 in the first place. I know it its just transverse loading, but I don't see how that makes A=0. We can discuss some more in M13.

Remember the general solution: $w = A\sin\sqrt{\dfrac{P}{EI}}x + B\cos\sqrt{\dfrac{P}{EI}}x + Cx + D$ only refers to

transverse (w) displacements. It does not tell us anything about axial displacements, u. Therefore if A=0, and hence w=0, it is still possible to have a non zero value of u.

In the notes describing mode, it says "higher modes only achieved if 1ˢᵗ mode is restrained" what does this mean? It means that to obtain the second or higher mode I would have to apply a pin at the center (or the 1/3 and 2/3 points) of the column. When I demonstrated the second mode I had to pinch the beam at the center to stop the first mode developing.

How do you impede the first mode to achieve the 2ⁿᵈ or 3ʳᵈ mode? See above.

What if n is not an integer for the loading ($P = \dfrac{n^2 \pi^2 EI}{L^2}$). What mode will the beam deform to? This is not possible, since if n does not equal an integer $\sin \sqrt{\dfrac{P}{EI}} L = 0$ is no longer true, and therefore the boundary conditions will not be satisfied.

What was it that you said deflected in the shape of an arc of a circle? Why is this different from column-sin displacements.   Very good question. A beam loaded by a pure moment will deflect into a circular arc, a conclusion we arrived at by symmetry considerations. In the case of a column there is also an axial force, so it is a different loading case. Also, we are dealing in all cases with small deflections, so the difference between a shallow circular arc and a shallow sinusoid are small.

If a beam is clamped or pinned at one end an axial force applied to the other end, will dw/dx=0 at x=L/2, like mentioned in the PRS.   Visually this looks to be the case but I can't think of a counter example. The answer depends on what boundary condition exists at the end where the loading is applied. We will cover a couple of relevant examples at the beginning of M13.

When you used the method of sections, why was the reaction force drawn parallel to the reaction force P at the pin joint? Shouldn't F be axial and thus it changes directionally (i.e shouldn't Fcosθ=P and not just F=-P? I guess that this doesn't change the moment equilibrium though. Good question. Remember we are dealing with small deflections and small rotations so Fcosθ=1, so this component doesn't matter. You could make the same comment about the rotation of the beam causing the shear force on a section to tilt out of the vertical. Since we are dealing with small deflections we have been assuming that the point of application and direction of action of the forces does not vary between the loaded and unloaded state.

Do we have to use the $w = e^{\lambda x}$ substitution or will we generally start at the $w = A\sin\sqrt{\dfrac{P}{EI}}x + B\cos\sqrt{\dfrac{P}{EI}}x + Cx + D$ equation? We will assume solutions of the form: $w = A\sin\sqrt{\dfrac{P}{EI}}x + B\cos\sqrt{\dfrac{P}{EI}}x + Cx + D$. This is more physically useful given the obvious sinusoidal shape of a buckled beam. I started with $w = e^{\lambda x}$ because I believe that this is how you would have been introduced to solutions of second order differential equations in 18.03 etc.

So we can't solve for A? Do we need more information. That is correct for the case of a perfect column. We will see in M13 that A becomes defined when we have an imperfect column.

If a load P is applied to a beam and it goes into 1st order buckling does it continue into 2nd order if P is increased? Or does the large P need to be applied outright? (Intuitively I can't imagine the beam suddenly developing a second buckle once already bent.) Your

intuition is absolutely correct. Once buckling occurs in a particular mode, it will stay in that mode.

So the deformation solution $w = A \sin \sqrt{\dfrac{P}{EI}}\, x + B \cos \sqrt{\dfrac{P}{EI}}\, x + Cx + D$ **doesn't take into account the fact that the beam deflects in the axial direction as well?** That is correct. It is not really a problem since the axial deflections are small compared to the bending deflections.

What determines how much buckling a structure can withstand? How does the structure fail? This will depend on the strength and toughness of the material of which the structure is made. We will start to discuss this in lecture M15.

What is the importance of critical buckling. Mathematically when will you get... It is important because it provides a useful upper bound on the actual buckling load. Obviously one will generally want to avoid buckling in the design of a structure as it represents a loss of load carrying capacity which compromises structural integrity.

Are buckling and vibrations of structures related to the beam's energy? …. You just talked about it!!!

But how do different buckling modes relate to this energy if first mode must be restrained? Are the other s really higher energy? Seems like they would be. They are!

Also if energy-related, why does the wave form seem continuous while buckling modes seem quantized for energy required to get there? Or were $P = \dfrac{n^2 \pi^2 EI}{L^2}$ **just barriers?** You need to distinguish two phenomena. Buckling, like vibrations of a string (guitar, violin etc), vibration of a beam or the vibration of air in an organ or wind instrument, will occur in characteristic modes (wavelengths). These are enforced by the boundary conditions that are applied. The column (like a string, the air in a pipe) is continuous and deforms in a continuous fashion. There are points (nodes) where there is no displacement which usually correspond to the boundary conditions or at rational fractions of the distance between boundaries.

The concise review at the beginning of class are very useful. Thank you – all criticism is helpful, positive or negative.

Yeah for filling the board before lecture!!

There were 5 muddy cards with no mud, or positive responses. And one suggestion for a useful lab experiment – which we may do in the next couple of lectures. Thank you.