Block 3 –Materials/Structural Integrity

Three purposes to this block of lectures:

1. To allow you to understand limits on stresses that a structure can carry: 1-D and multiaxial – in order to design
2. To allow you to understand the origins (and limits) of the strength in the different classes of material –identifying “unobtainium”, origins and limitations on models for strength
3. To allow you to select materials for specific functions, understand design process.

Note: “Failure” occurs when a structure cannot meet its design requirements. Does not necessarily correspond to material “failure” – e.g buckling. We will focus on material aspects here.

Materials can behave in a variety of ways according to the loading. Examples:

1. a piece of metal wire (paper clip):

Can “twang” it – elastic deformation
Bend it - leading to permanent deformation
Cyclic loading – rupture into two pieces (fatigue)

2. A piece of aluminum foil:

Load it up and it “necks down” before fracture
Put a notch in it and it tears

3. A balloon (pressurized aircraft fuselage)

Stick a pin in it and it explodes – catastrophic failure

4. e.g piece of solder wire with a weight hanging from it under a bright light - creep

We will examine four types of failure in Unified; Yield (ductile failure), Fracture (brittle failure), Fatigue (cyclic loading) and creep (time/temperature dependent failure) – necessarily brief. Read Ashby and Jones for more details.

M15 Yielding and Plasticity

Reading: Ashby and Jones ch. 8, 11

Characteristic stress-strain curve and material response for a ductile metal under uniaxial loading

Load-extension curve for a bar of ductile metal (e.g. annealed copper) pulled in tension.

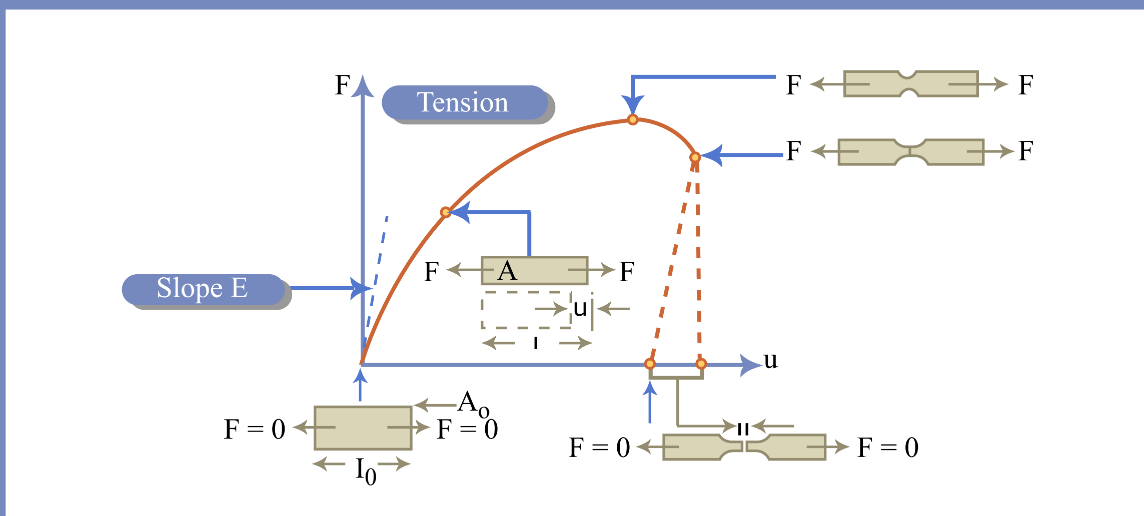


Image adapted from: Ashby, M. F., and D. R. H. Jones. *Engineering Materials 1*. Pergamon Press, 1980.

Unloading and reloading is elastic, permanent strain:

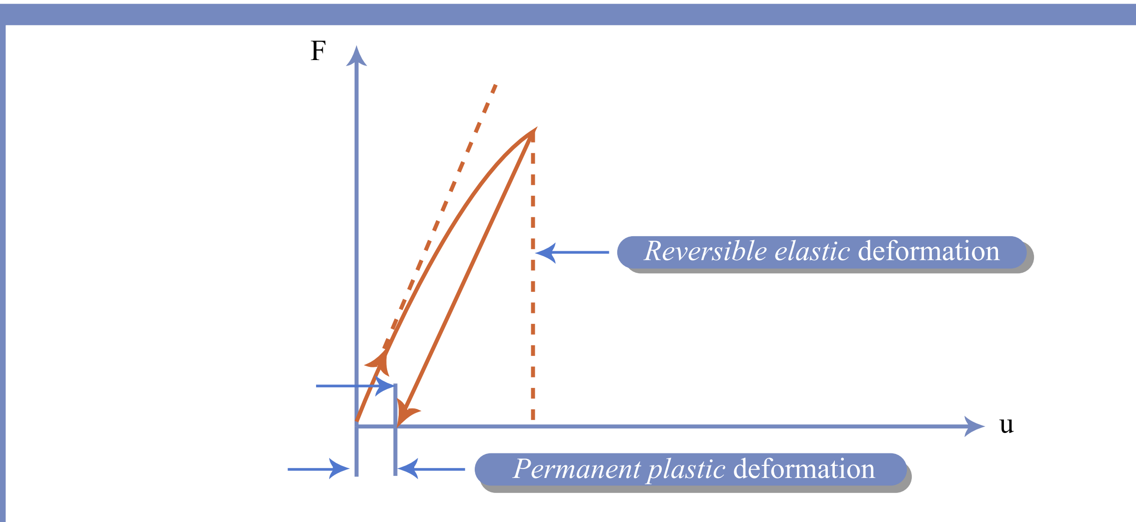


Image adapted from: Ashby, M. F., and D. R. H. Jones. *Engineering Materials 1*. Pergamon Press, 1980.

Most engineering alloys have an initial linear elastic region followed by yield and non-linear plasticity:

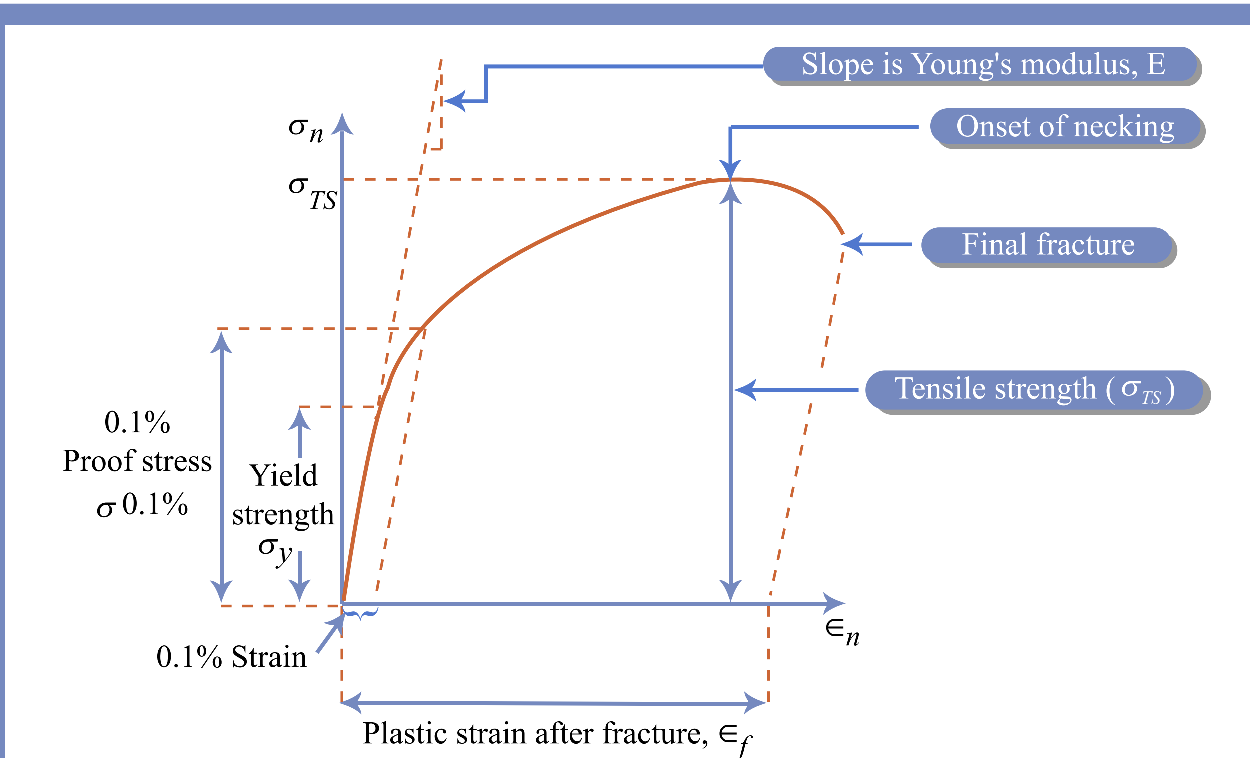


Image adapted from: Ashby, M. F., and D. R. H. Jones. *Engineering Materials 1*. Pergamon Press, 1980.

Key features of stress-strain response:

Yield stress, σ_y
Tensile strength σ_{ts}
Permanent strain after unloading
Linear elastic unloading-reloading
Work hardening (increasing yield stress with increasing plastic strain).

Up to now we have not distinguished between compressive and tensile response of materials. Need to be careful with large plastic strains:

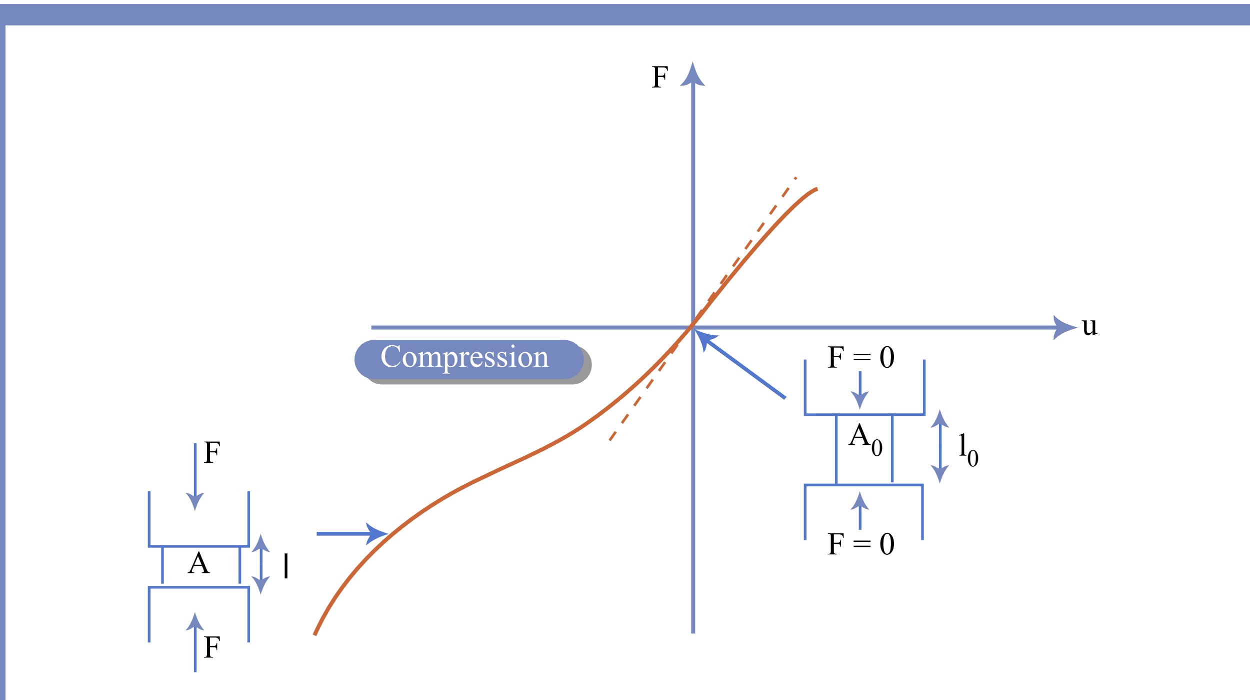


Image adapted from: Ashby, M. F., and D. R. H. Jones. *Engineering Materials 1*. Pergamon Press, 1980.

Apparent difference between compression and tension due to increase in cross-sectional area in compression vs. reduction in tension. Modify definitions of stress and strain to account for this:

True stress, σ_t:

Load/actual cross-sectional area: σ_t=F/A (vs $\sigma_{nominal}$ =F/A_0)

True strain, ε_t defined incrementally:

$$\delta\varepsilon = \frac{\delta u}{l} = \frac{\delta l}{l} \Rightarrow d\varepsilon = \frac{dl}{l}$$

Hence strain developed from an initial length l to a final length l_0 is given by:

$$\varepsilon_t = \int_{l_0}^{l} \frac{dl}{l} = \ln\left(\frac{l}{l_0}\right)$$

Also, in a plastic deformation volume is conserved (material is incompressible). Hence, if the elastic deformation is negligible, $A = \frac{A_0 l_0}{l}$. Hence,

$$\sigma_t = \frac{F}{A} = \frac{Fl}{A_0 l_0}$$

Rearranging we obtain:

$$\sigma_t = \sigma_n(1+\varepsilon_n)$$

$$\varepsilon_t = \ln(1+\varepsilon_n)$$

Replot stress-strain curves, appear symmetric:

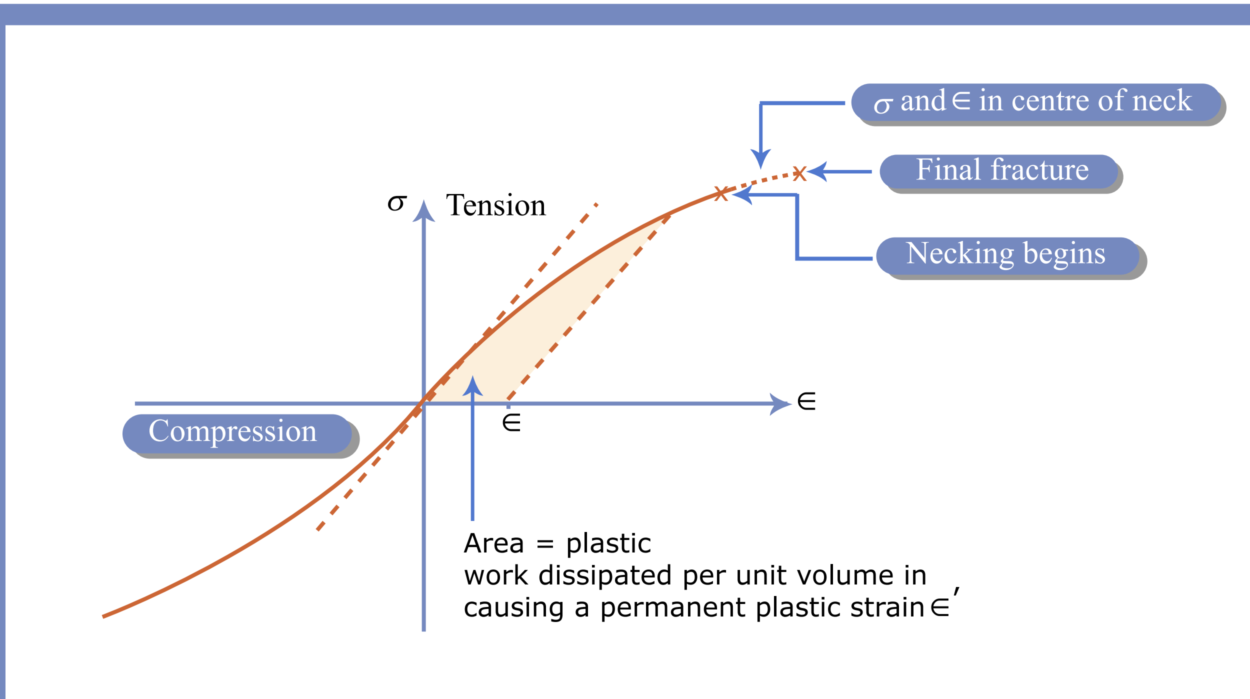


Image adapted from: Ashby, M. F., and D. R. H. Jones. *Engineering Materials 1*. Pergamon Press, 1980.

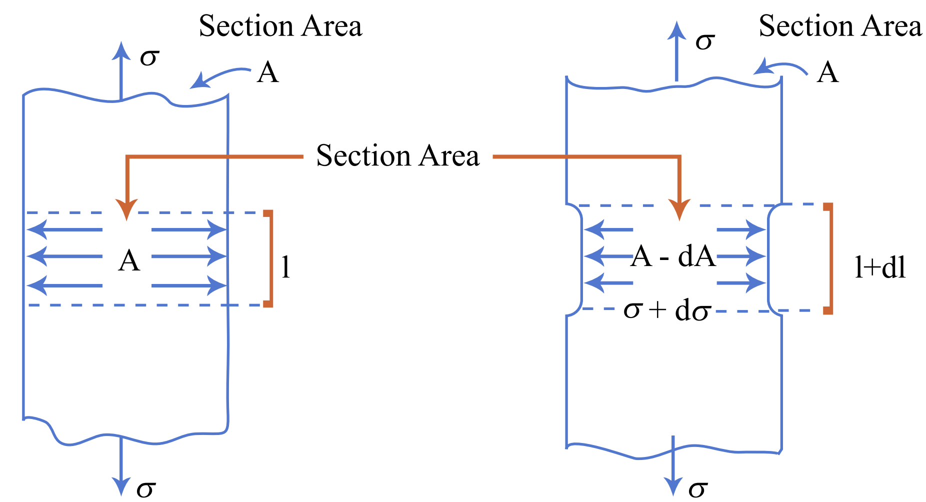
Work is done causing plastic deformation – it is an irreversible process:

$$U_{pl} = \int \sigma d\varepsilon - \frac{\sigma^2}{2E}$$

Explains why tools, metal heat up during metal working. Also why frictional sliding generates heat

"Necking" – tensile instability

Can examine onset of necking using true-stress-true strain definitions:



The formation of a neck in a bar of material which is being deformed plastically.

Image adapted from: Ashby, M. F., and D. R. H. Jones. *Engineering Materials 1*. Pergamon Press, 1980.

From tensile nominal stress- nominal strain curve

know that: $A\sigma_t = F = \text{constant}$

Then $Ad\sigma_t + \sigma_t dA = 0$

Or: $\dfrac{d\sigma_t}{\sigma_t} = -\dfrac{dA}{A}$

But volume is conserved during plastic flow, so:

$$-\frac{dA}{A} = \frac{dl}{l} = d\varepsilon_t$$

$$\frac{d\sigma_t}{\sigma_t} = d\varepsilon_t \Rightarrow \frac{d\sigma_t}{d\varepsilon_t} = \sigma_t$$

Substituting for nominal stresses and strains:

$$\frac{d\sigma_n}{d\varepsilon_n} = 0$$

The necking point is important because it defines a limit on how thin one can roll or draw metal sheets/foils or draw wires in a single pass.

Materials can have a wide range of values of yield stress and ultimate tensile strength:

Yield Strength, σ_y, Tensile Strength, σ_{TS}, and Tensile Ductility, ϵ_f

Material	σ_y/MN m^{-2}	σ_{TS}/MN m^{-2}	ϵ_f
Diamond	50000	–	0
Silicon carbide, SiC	10000	–	0
Silicon nitride, Si_3N_4	8000	–	0
Silica glass, SiO_2	7200	–	0
Tungsten carbide, WC	6000	–	0
Niobium carbide, NbC	6000	–	0
Alumina, AI_2O_3	5000	–	0
Beryllia, BeO	4000	–	0
Mullite	4000	–	0
Titanium carbide, TiC	4000	–	0
Zirconium carbide, ZrC	4000	–	0
Tantalum carbide, TaC	4000	–	0
Zirconia, ZrO_2	4000	–	0
Soda glass (standard)	3600	–	0
Magnesia, MgO	3000	–	0
Cobalt and alloys	180-2000	500-2500	0.01-6
Low-alloy steels (water-quenched and tempered)	500-1980	680-2400	0.02-0.3
Pressure-vessel steels	1500-1900	1500-2000	0.3-0.6
Stainless steels, austenitic	286-500	760-1280	0.45-0.65
Boron/epoxy composites (tension-compression)	–	725-1730	–
Nickel alloys	200-1600	400-2000	0.01-0.6
Nickel	70	400	0.65
Tungsten	1000	1510	0.01-0.6
Molybdenum and alloys	560-1450	665-1650	0.01-0.36
Titanium and alloys	180-1320	300-1400	0.06-0.3
Carbon steels (water-quenched and tempered)	260-1300	500-1880	0.2-0.3
Tantalum and alloys	330-1090	400-1100	0.01-0.4
Cast irons	220-1030	400-1200	0-0.18
Copper alloys	60-960	250-1000	0.01-0.55
Copper	60	400	0.55
Cobalt/tungsten carbide cermets	400-900	900	0.02
CFRPs (tension-compression)	–	670-640	–
Brasses and bronzes	70-640	230-890	0.01-0.7
Aluminium alloys	100-627	300-700	0.05-0.3
Aluminium	40	200	0.5
Stainless steels, ferritic	240-400	500-800	0.15-0.25
Zinc alloys	160-421	200-500	0.1-1.0
Concrete, steel reinforced (tension or compression)	–	410	0.02
Alkali halides	200-350	–	0
Zirconium and alloys	100-365	240-440	0.24-0.37
Mild steel	220	430	0.18-0.25
Iron	50	200	0.3
Magnesium alloys	80-300	125-380	0.06-0.20
GFRPs	–	100-300	–
Beryllium and alloys	34-276	380-620	0.02-0.10
Gold	40	220	0.5
PMMA	60-110	110	0.03-0.05
Epoxies	30-100	30-120	–
Polyimides	52-90	–	–

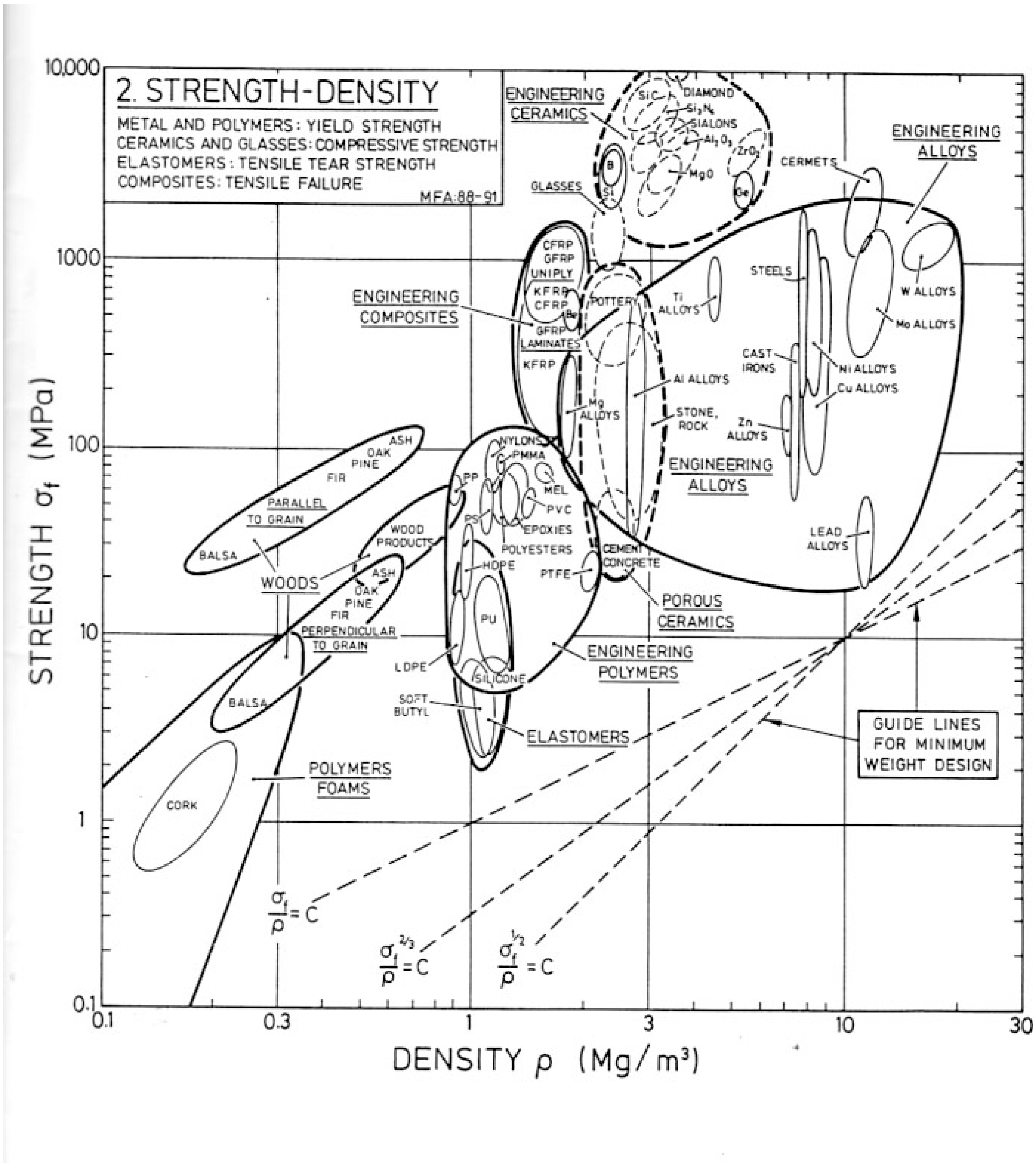
Image adapted from: Ashby, M. F., and D. R. H. Jones. *Engineering Materials 1*. Pergamon Press, 1980.

Continued...

Material	σ_y/MN m^{-2}	σ_{TS}/MN m^{-2}	ϵ_f
Nylons	49-87	100	–
Ice	85	–	0
Pure ductile metals	20-80	200-400	0.5-1.5
Polystyrene	34-70	40-70	–
Silver	55	300	0.6
ABS/polycarbonate	55	60	–
Common woods (compression, ‖ to grain)	–	35-55	–
Lead and alloys	11-55	14-70	0.2-0.8
Acrylic/PVC	45-48	–	–
Tin and alloys	7-45	14-60	0.3-0.7
Polypropylene	19-36	33-36	–
Polyurethane	26-31	58	–
Polyethylene, high density	20-30	37	–
Concrete, non-reinforced, compression	20-30	–	0
Natural rubber	–	30	5.0
Polyethylene, low density	6-20	20	–
Common woods (compression, ⊥ to grain)	–	4-10	–
Ultrapure f.c.c. metals	1-10	200-400	1-2
Foamed polymers, rigid	0.2-10	0.2-10	0.1-1
Polyurethane foam	1	1	0.1-1

Image adapted from: Ashby, M. F., and D. R. H. Jones. *Engineering Materials 1*. Pergamon Press, 1980.

As for elastic properties, can group according to classes of material:



Ashby, Material Selection

Note range of strengths for alloy of a particular metal.

Two questions:

1. How can we control, improve strength of metals

2. How do we design for strength in cases where we have multiaxial loading (torsion, web of beams, fuselages)

Next time: Look at mechanisms of plastic deformation, understand strengthening mechanisms, understand physical basis for multi-axial yield criteria.

Work of deformation

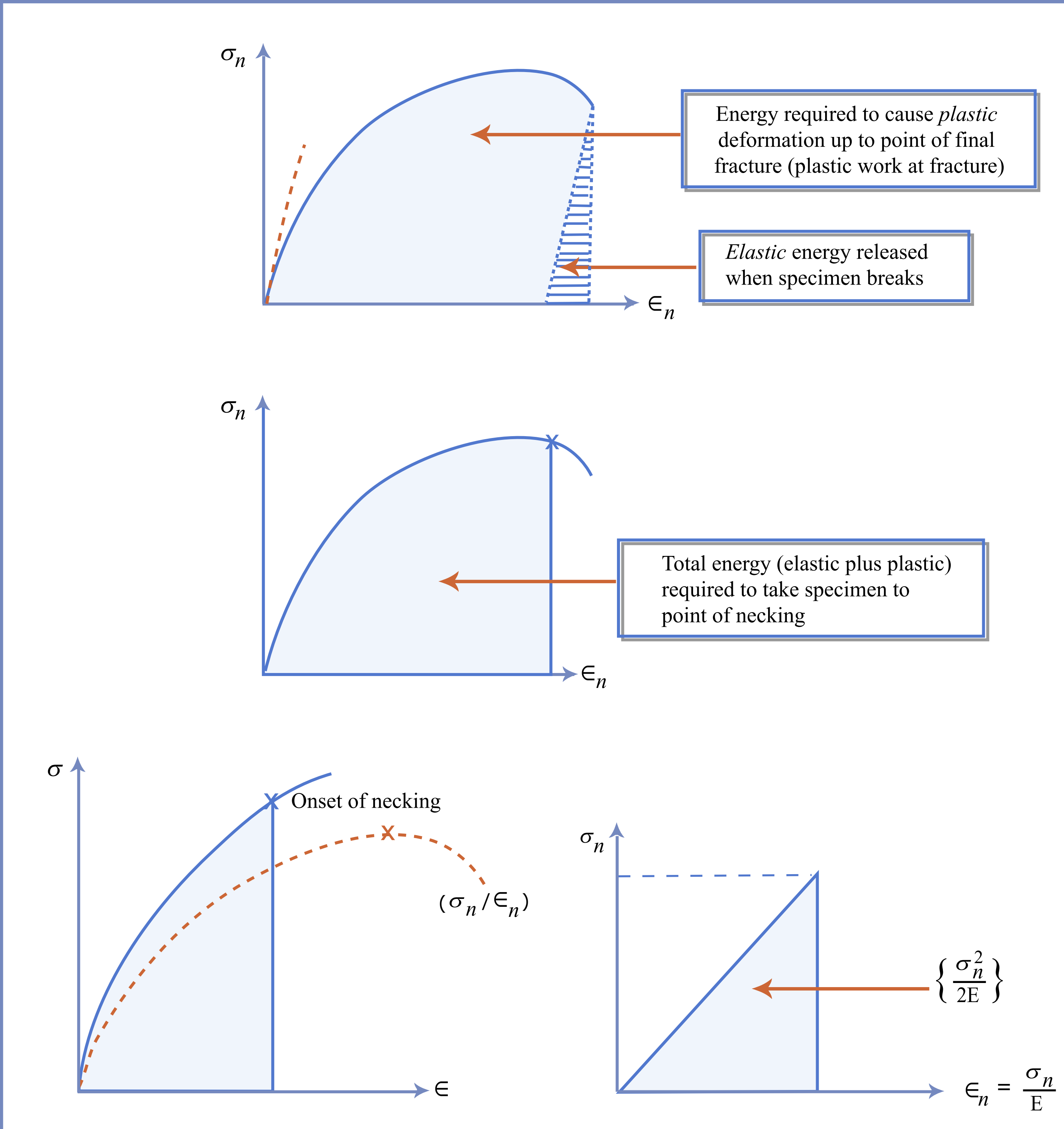


Image adapted from: Ashby, M. F., and D. R. H. Jones. *Engineering Materials 1*. Pergamon Press, 1980.

M16 Origins of Plasticity, Alloying

Reading: Ashby and Jones ch. 9, 10

Objective: understand what governs yield and plasticity and what we can do to improve it.

Suspect that this might originate at the atomic scale: bonding, atomic packing.

From last term we had:

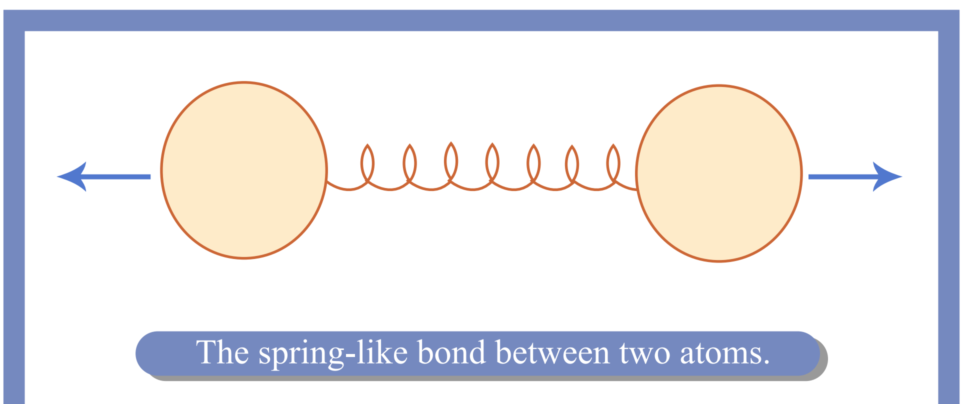
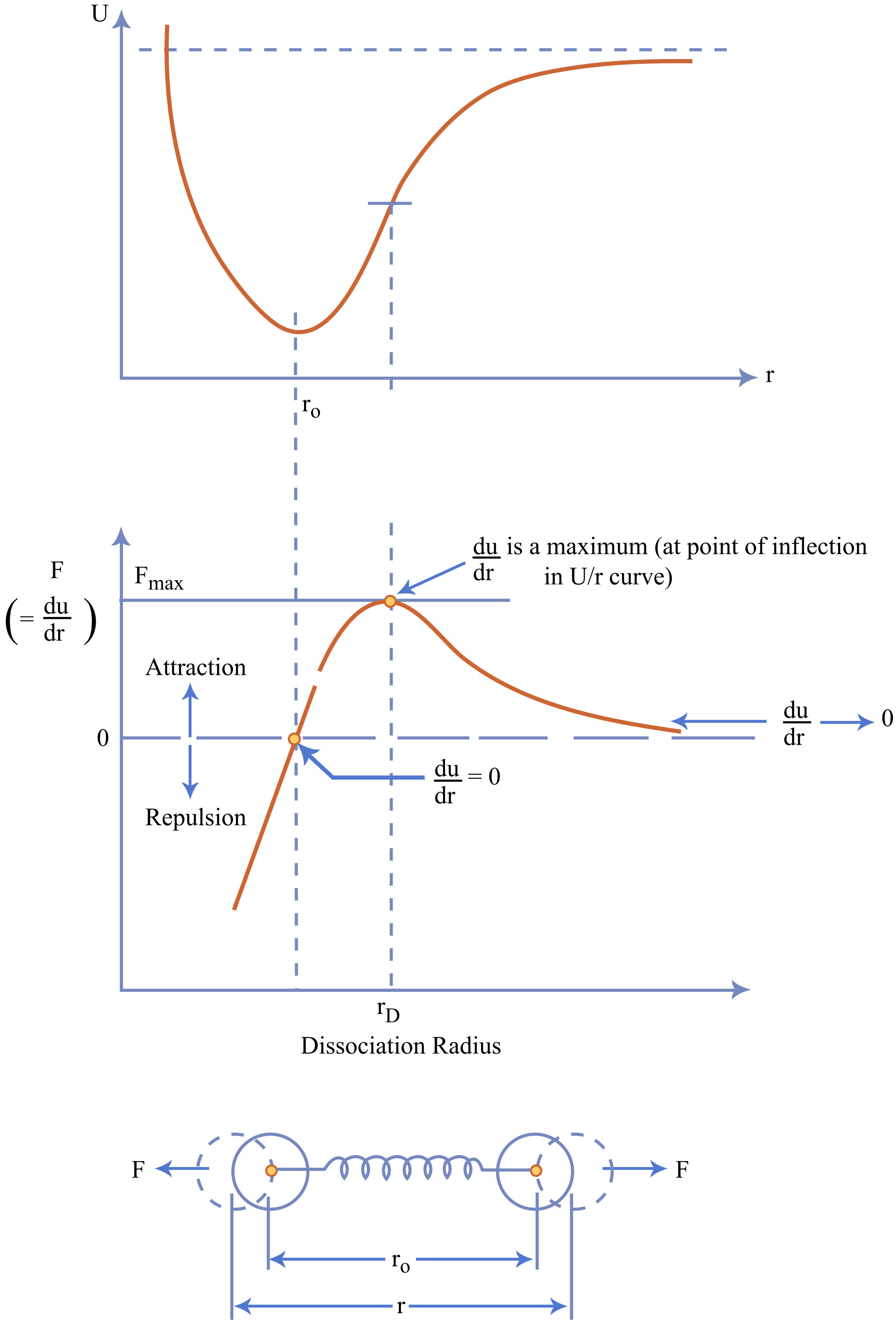


Image adapted from: Ashby, M. F., and D. R. H. Jones. *Engineering Materials 1*. Pergamon Press, 1980.

$$U(r) = -\frac{A}{r^m} + \frac{B}{r^n} \quad (m < n) \text{ and } F = \frac{dU}{dr}$$



The energy curve (top), when differentiated gives the force-distance curve (centre).

Image adapted from: Ashby, M. F., and D. R. H. Jones. *Engineering Materials 1*. Pergamon Press, 1980.

Might expect strength to be linked to maximum in the atomic force-displacement curve, i.e.:

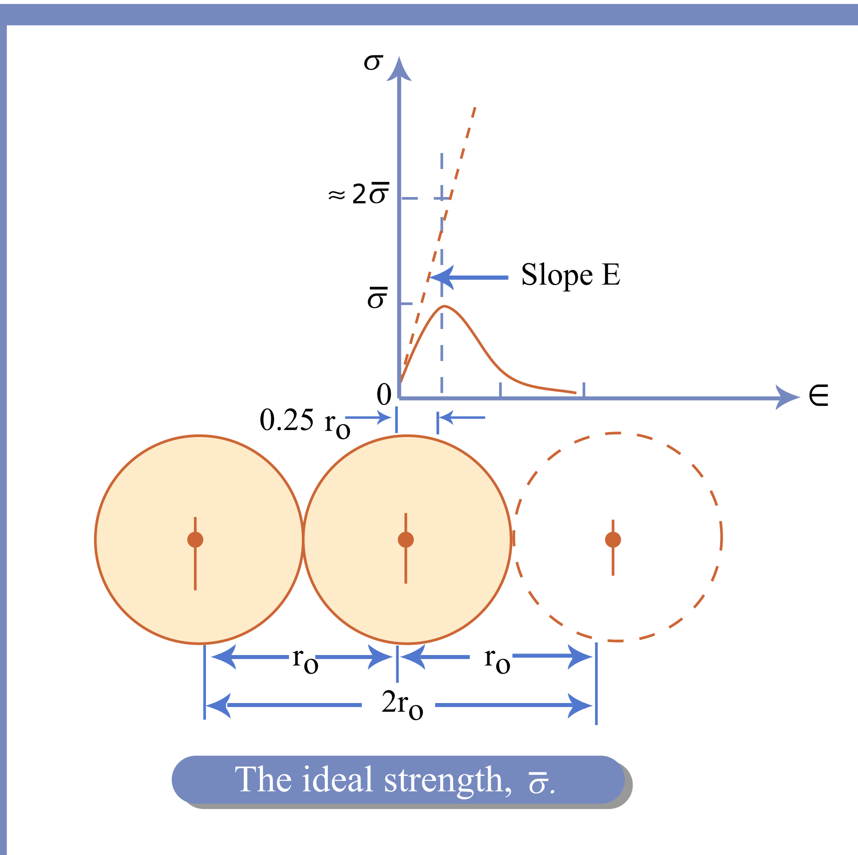
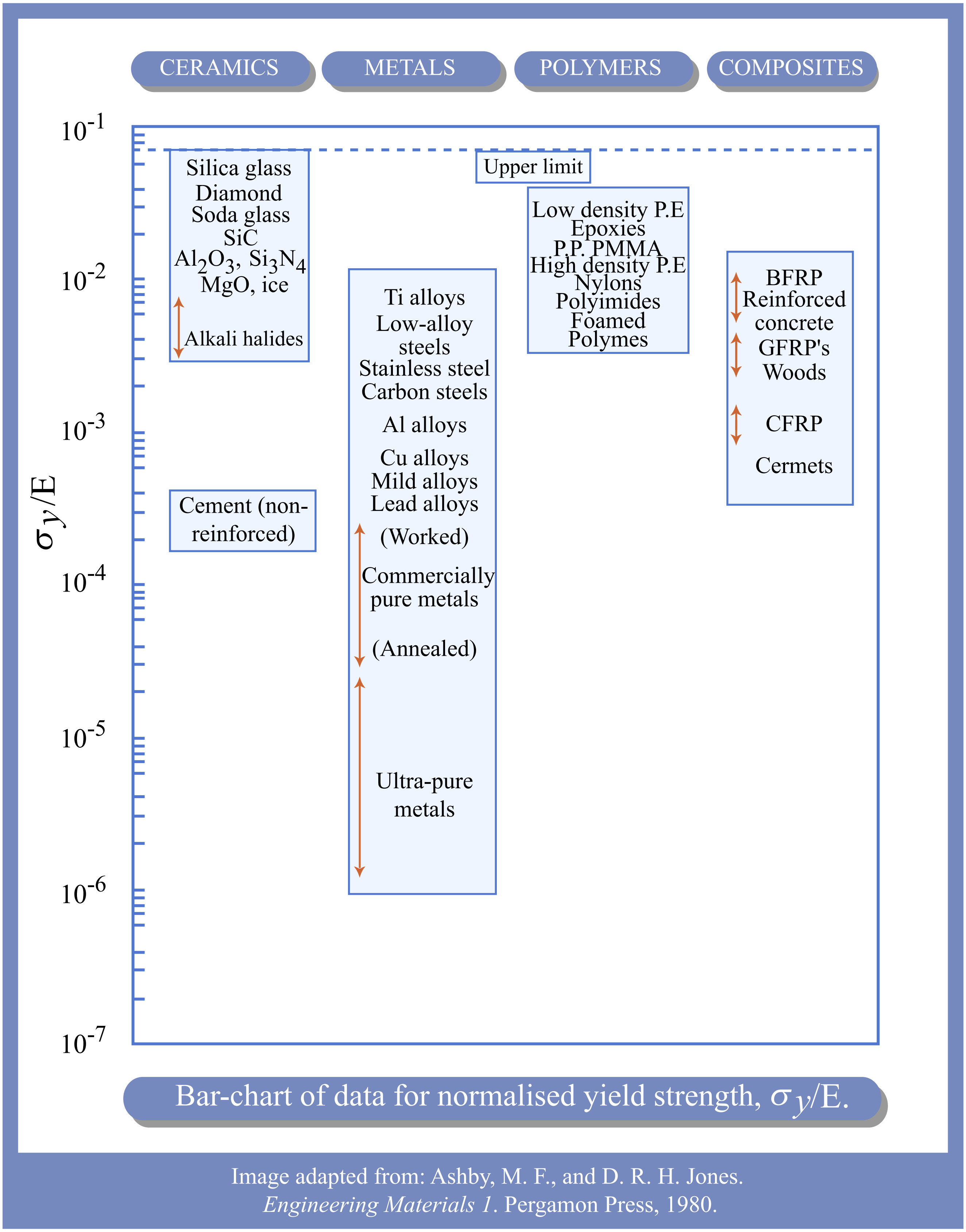


Image adapted from: Ashby, M. F., and D. R. H. Jones. *Engineering Materials 1*. Pergamon Press, 1980.

This provides estimates for yield stress:

$$\frac{E}{30} \leq \sigma_y \leq \frac{E}{4}$$

In reality, actual yield stresses, particularly for metals, are much lower than these estimates:


CERAMICS
METALS
POLYMERS
COMPOSITES
10-1
10-2
10-3
10-4
10-5
10-6
10-7
σy/E
Upper limit
Silica glass
Diamond
Soda glass
SiC
Al2O3, Si3N4
MgO, ice
Alkali halides
Cement (non-reinforced)
Ti alloys
Low-alloy steels
Stainless steel
Carbon steels
Al alloys
Cu alloys
Mild alloys
Lead alloys
(Worked)
Commercially pure metals
(Annealed)
Ultra-pure metals
Low density P.E
Epoxies
P.P. PMMA
High density P.E
Nylons
Polyimides
Foamed
Polymes
BFRP
Reinforced concrete
GFRP's
Woods
CFRP
Cermets
Bar-chart of data for normalised yield strength, σy/E.
Image adapted from: Ashby, M. F., and D. R. H. Jones.
Engineering Materials 1. Pergamon Press, 1980.

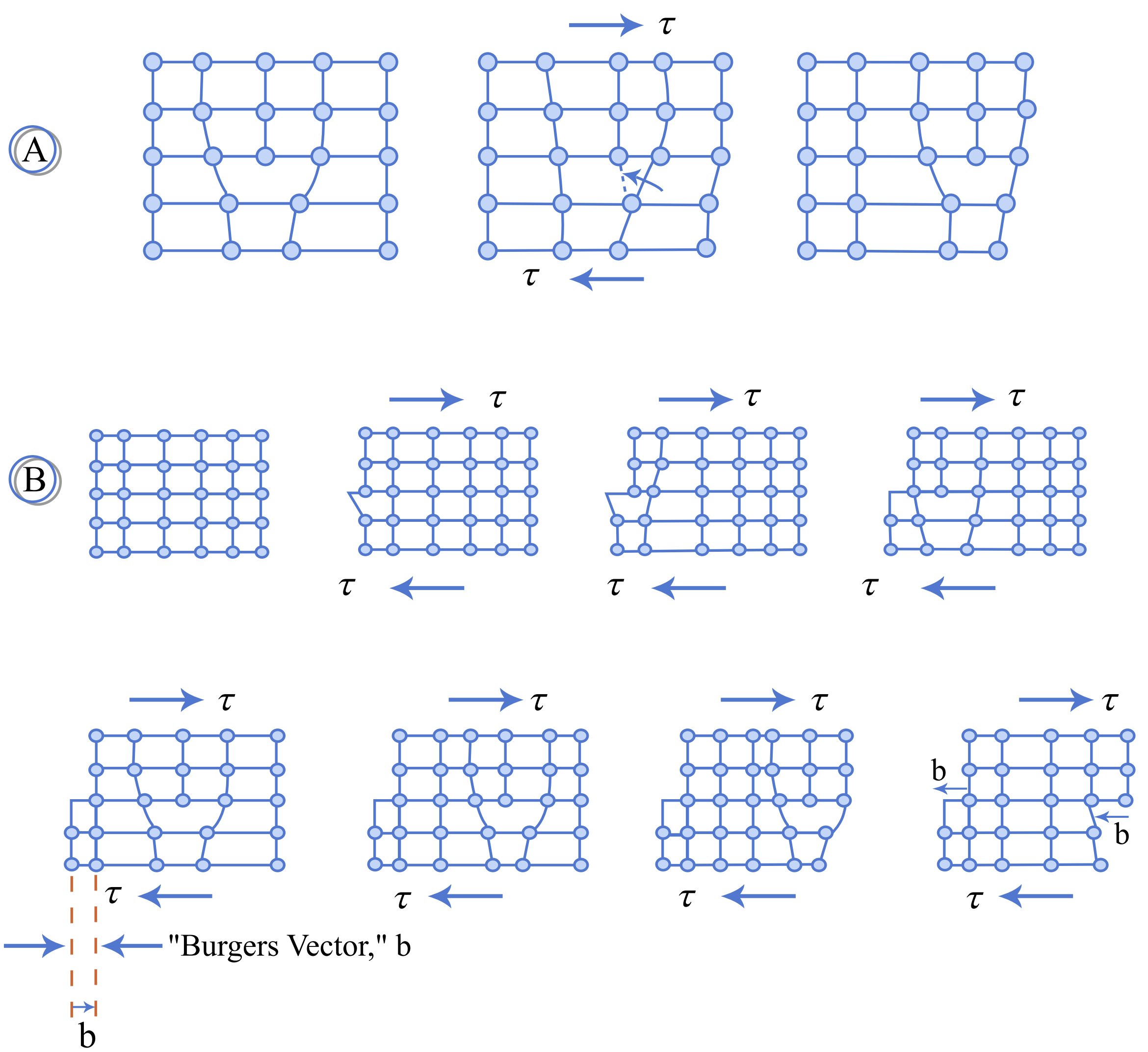
Why?

Dislocations

Recall: metals are made of grains with atoms in regular repeating crystal lattices. Contain defects.

A dislocation is an extra half plane of atoms inserted into the lattice.

How does this affect yield? Consider a shear stress applied to a lattice containing a dislocation:

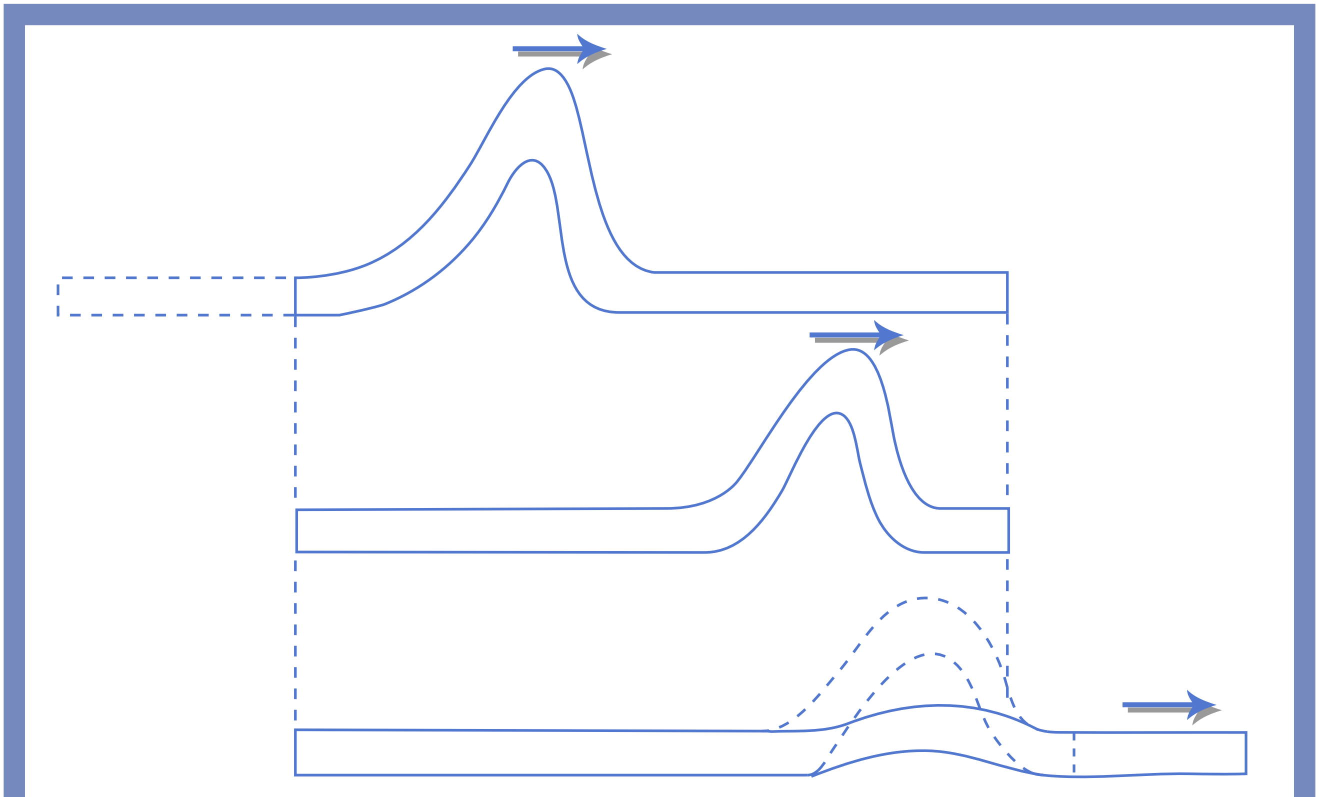


How an edge dislocation moves through a crystal:
A: Shows how the atomic bonds at the centre of the dislocation break and reform to allow the dislocation to move.
B: Shows a complete sequence for the introduction of a dislocation into a crystal from the left-hand side, its migration through the crystal, and its expulsion on the right-hand side; this process causes the lower half of the crystal to slip by a distance b under the upper half.

Image adapted from: Ashby, M. F., and D. R. H. Jones. *Engineering Materials 1*. Pergamon Press, 1980.

Deformation occurs one atom, bond at a time.

Analogue to moving a carpet by pushing rucks across a floor:



The 'carpet-ruck' analogy of an edge dislocation.

Image adapted from: Ashby, M. F., and D. R. H. Jones. *Engineering Materials 1*. Pergamon Press, 1980.

Have discussed edge dislocations, but also “screw” disclocations:

Much more we could say about dislocations, but with our simple model of an edge dislocation we can understand some key facts that will help us understand strengthening mechanisms and alloying.

Key facts about dislocations:

1. Move under action of shear stresses
2. Controlling dimension is the burger vector, b
3. Move (glide) on slip planes – planes between close packed planes have lower densities of bonds – less resistance to shear
4. Dislocations are line defects – long in one direction
5. Local distortion of crystal lattice in transverse directions stretches bonds, stores strain energy.
6. In order to minimize strain energy dislocations will try to straighten out (think of rubber bands)

Can estimate line tension by considering strain energy associated with dislocation core

Strains at core ≈ 0.5 Therefore stresses $\approx 0.5G$ (G = shear modulus)

Volume of material at dislocation core $\approx \pi b^2$

Strain energy per unit volume in core

$$\approx \frac{1}{2}\sigma\varepsilon = \frac{G}{8}$$

Therefore line tension, $T = \dfrac{\pi G b^2}{8} \approx \dfrac{G b^2}{2}$

Force per unit length = energy per area (1-D version of the surface tension of a liquid's surface). Dislocations tend to be straight.

Strengthening Methods for Metals

Strengthening methods for metals can be largely understood in terms of increasing the resistance to dislocation motion.

A crystal yields when the force τb (per unit length) exceeds, f, the resistance to motion:

$$\tau_y = \frac{f}{b} \qquad \tau_y \text{ The shear yield stress}$$

f can arise from several contributions

1. Solid solution strengthening (alloying) eg. Zn in Cu to make brass, 5000 series Al alloys (Mg)

Zn atoms are larger than Cu and distort the lattice, impede dislocation motion, “roughen” the slip planes

$$\tau_y \propto c^{1/2} \quad \text{where c = conc. of alloying element}$$

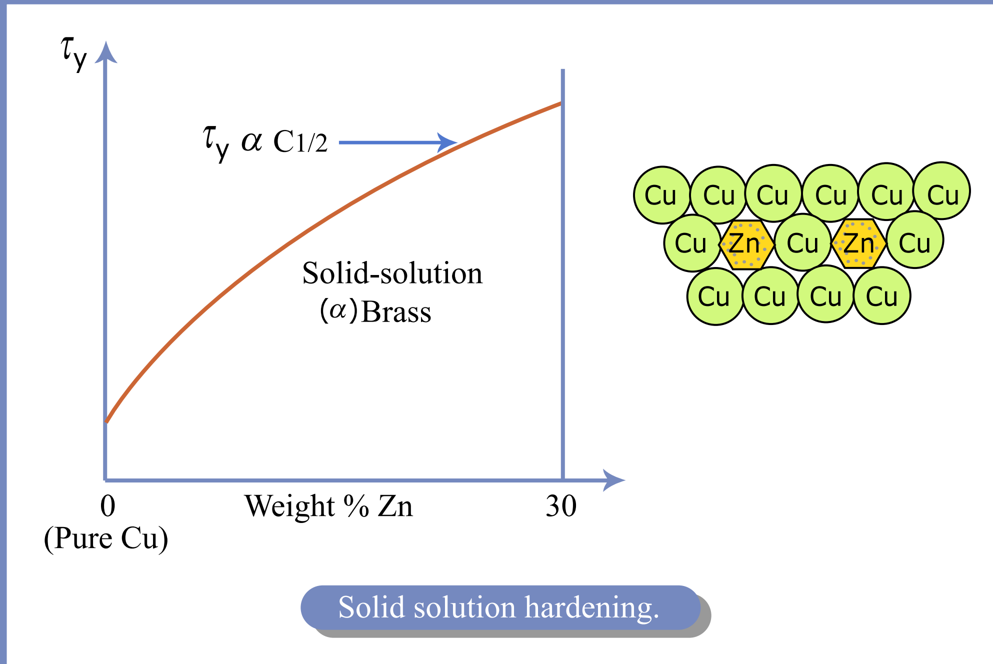


Image adapted from: Ashby, M. F., and D. R. H. Jones. *Engineering Materials 1*. Pergamon Press, 1980.

2. Precipitate and dispersion strengthening: small, hard precipitate particles form from an alloy solution, or small, hard particles separately introduced into alloy. Particles act to pin dislocations:

Strengthening given by: $\tau_y = \dfrac{2T}{bL}$ where L = particle spacing.

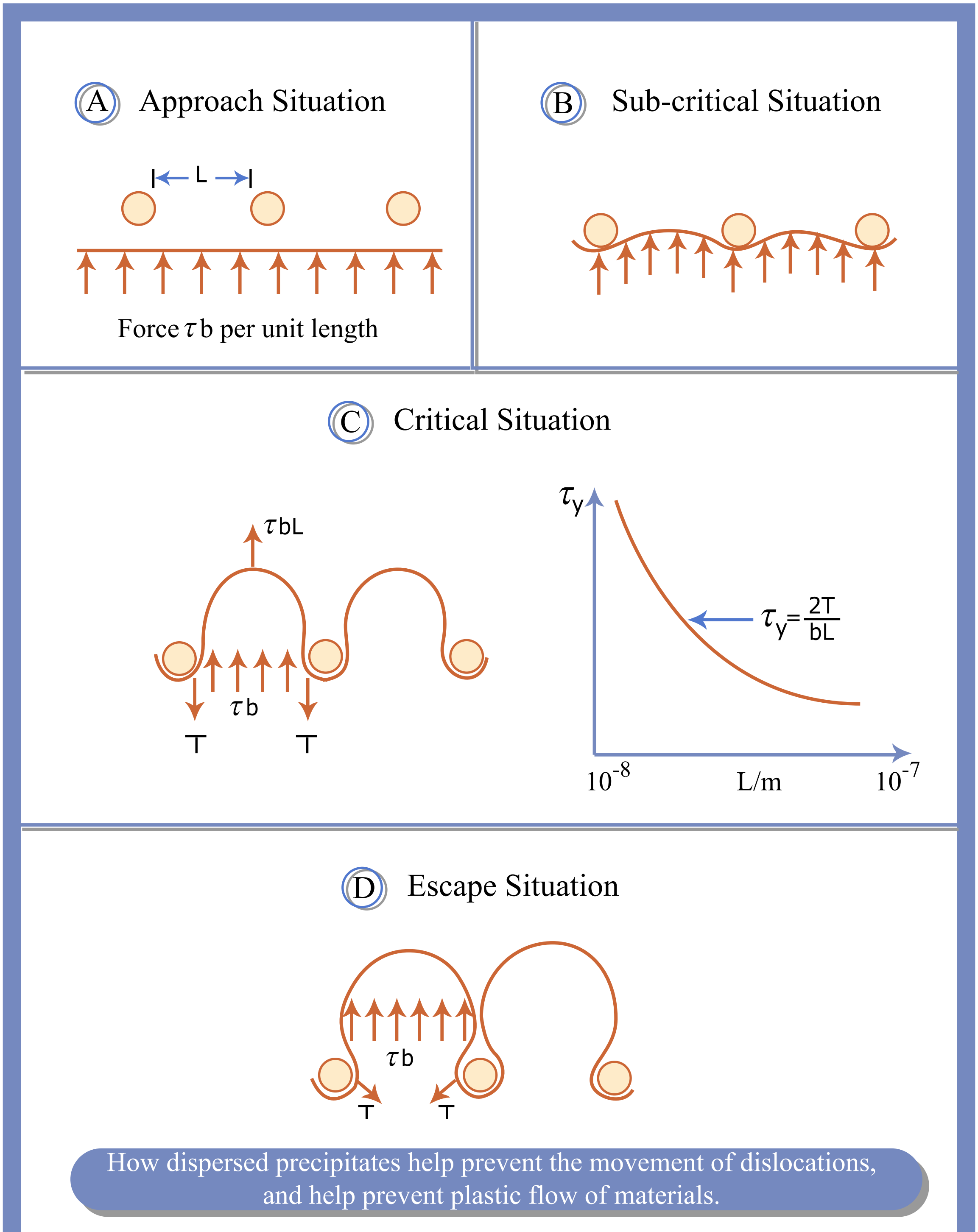
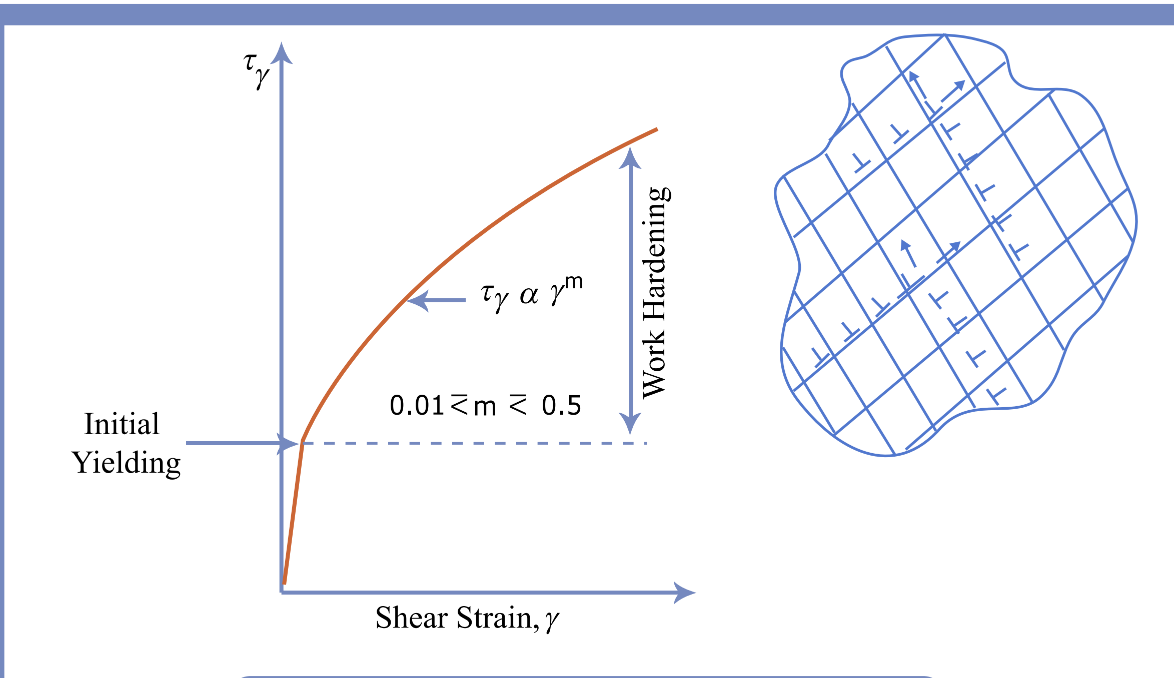


Image adapted from: Ashby, M. F., and D. R. H. Jones. *Engineering Materials 1*. Pergamon Press, 1980.

Principal strengthening mechanisms in most steels, Al 2000 series (Cu), Al 6000 series (Mg/Si) and Al 7000 series (Zn, Mg/Cu/Mn). In 2000 series $CuAl_2$ intermetallic precipitates provide strengthening. 2000 and 7000 series aluminum also called "age hardening" alloys – precipitate particles grow with time and temperature. Must be careful to define usage temperatures, also processing temperatures (welding can be problematic).

3. Work hardening.

Dislocations interfere with each other – impede motion, increase strength:



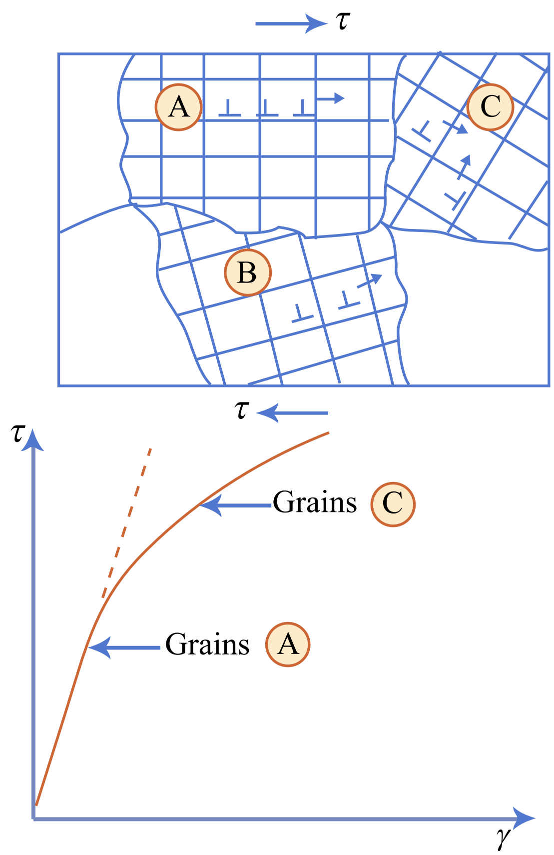
Collision of dislocations leads to work-hardening.

Image adapted from: Ashby, M. F., and D. R. H. Jones. *Engineering Materials 1*. Pergamon Press, 1980.

Rolling metal sheet, bars at room temperature increases strength by work-hardening.

4. Grain boundaries

Grain boundaries act as obstacles to dislocation motion. Depending on orientation relative to applied stress grains will yield at different stresses.



The progressive nature of yielding in a polycrystalline material.

Image adapted from: Ashby, M. F., and D. R. H. Jones. *Engineering Materials 1*. Pergamon Press, 1980.

Typically $\tau_{y(\text{polycrystal})} = 1.5\tau_{y(\text{single crystal})}$

In general structural metals exploit all four strengthening mechanisms to varying degrees.

Next time: Yield under multi-axial loading.

M17 Yield of Structures

Reading: Crandall, Dahl and Lardner 3.11

In general structures carry multiaxial stresses. Thus far we have looked at the phenomena and mechanisms of yield under uniaxial loading (tensile stress-strain and dislocations moving under shear stresses).

Key idea, backed up by experimental observations, Yield is caused by shear stresses – cause dislocations to move.

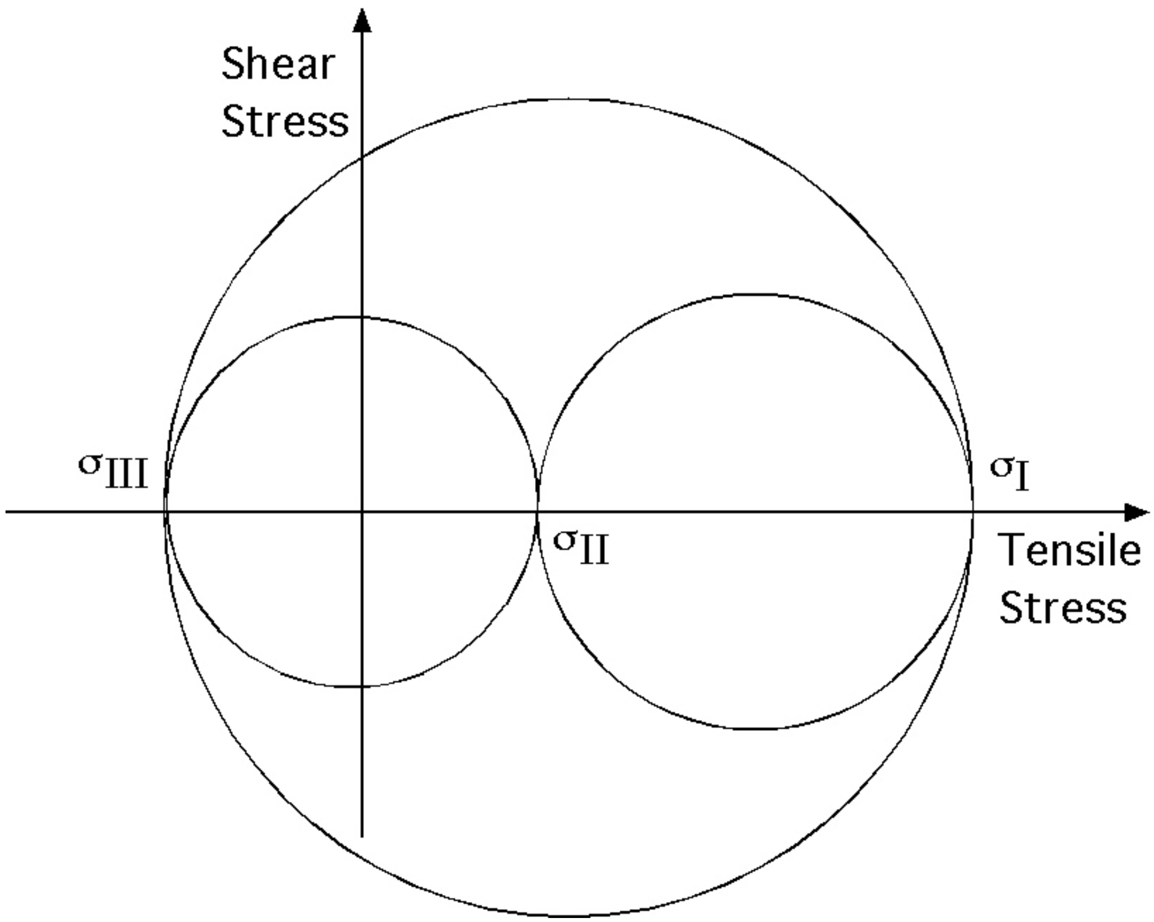
Two yield criteria have been proposed to exploit this observation:

1. Tresca's yield criterion

   Yield occurs if maximum shear stress exceeds the shear yield stress, τ_y. Therefore need to be able to calculate maximum shear stress in an arbitrary 3-

D stress state. Proceed using Mohr's circle, tensors, eigenvalues of stress matrix.

Useful to represent as Mohr's circles for 3-D stress state on single diagram:



Principal stresses: $\sigma_I, \sigma_{II}, \sigma_{III}$

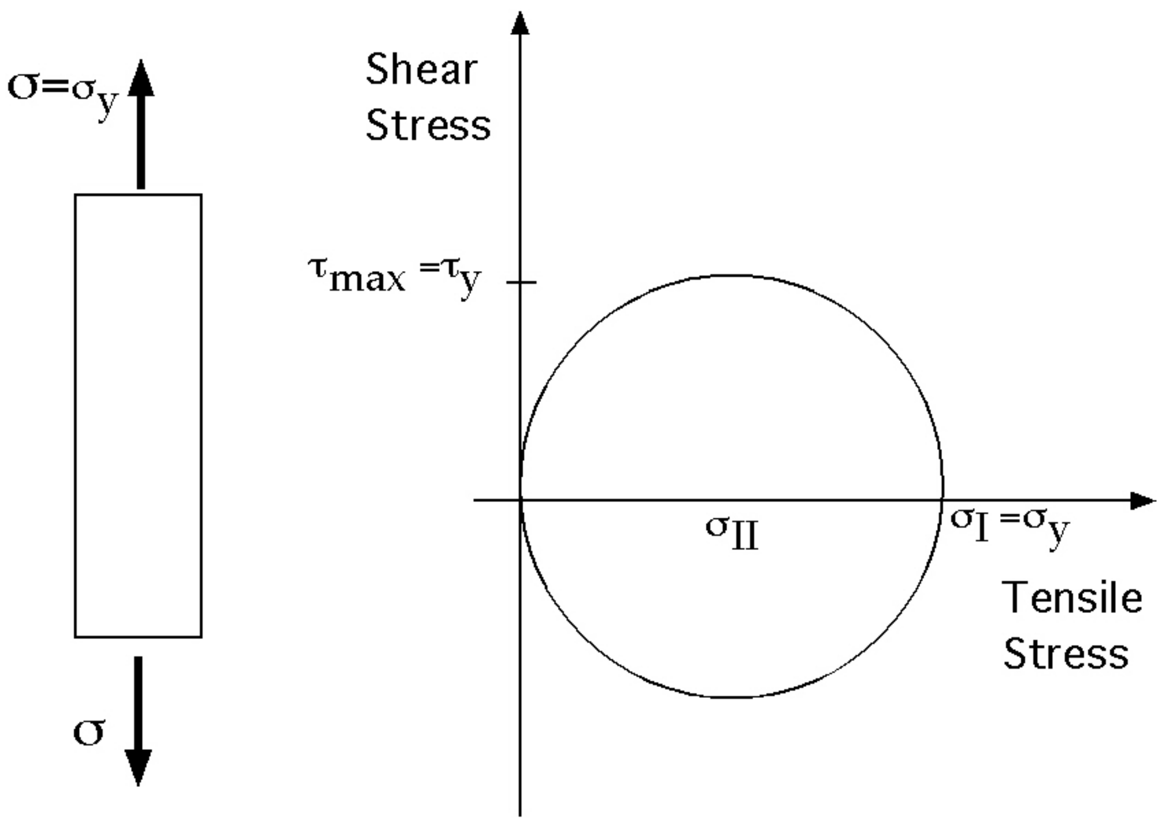
Maximum shear stresses

$= \frac{1}{2}$ (maximum difference between principal stresses)

So yield occurs when

$$\frac{|\sigma_I - \sigma_{II}|}{2} \text{ or } \frac{|\sigma_{II} - \sigma_{III}|}{2} \text{ or } \frac{|\sigma_I - \sigma_{II}|}{2} \geq \tau_y$$

Can relate to the uniaxial tension test:



yield when $\frac{|\sigma_y - 0|}{2} = \tau_y$ i.e

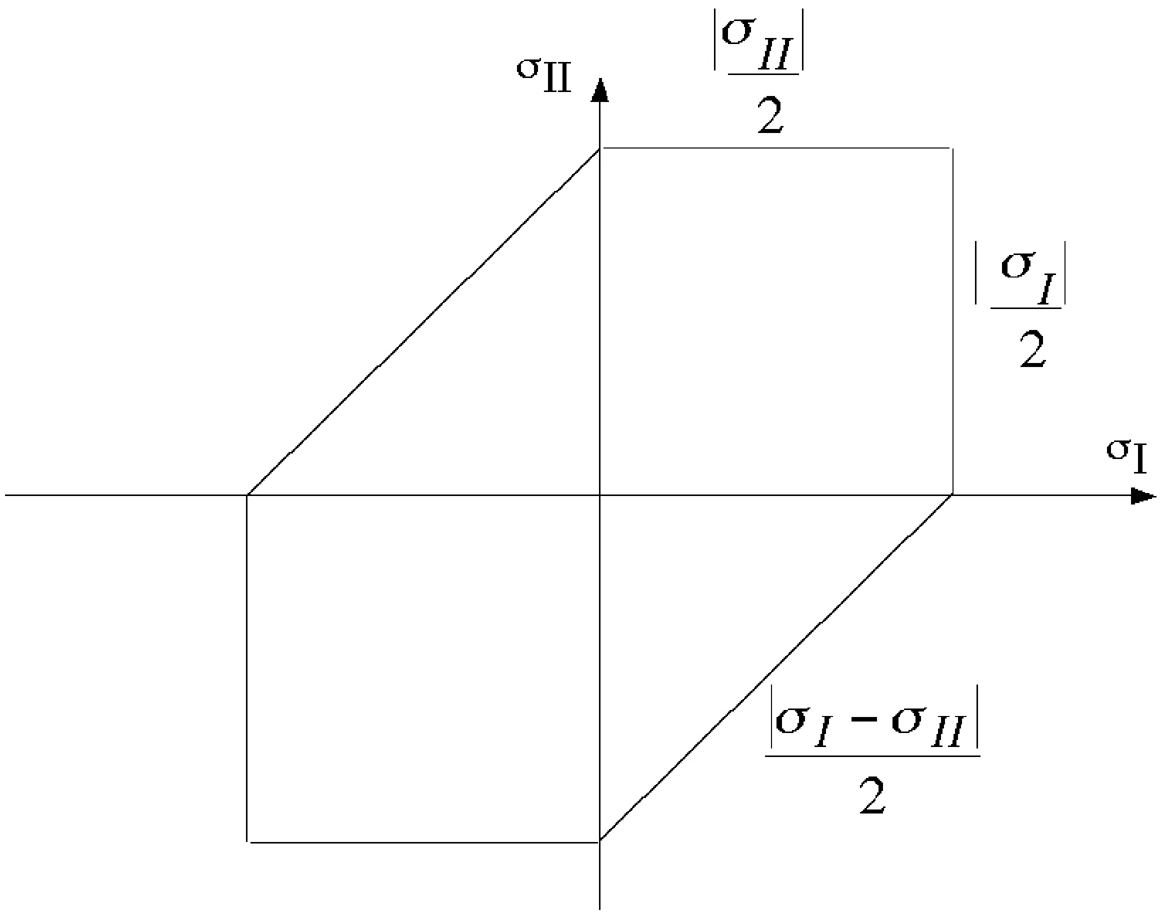
$$\tau_y = \frac{\sigma_y}{2}$$ - Tresca shear yield stress

Consider plane stress case: $\sigma_{III} = 0$

So yield occurs when

$$\tau_y = \frac{|\sigma_I - \sigma_{II}|}{2} \text{ or } \tau_y = \frac{|\sigma_I|}{2} \text{ or } \tau_y = \frac{|\sigma_{II}|}{2}$$

Can plot as a failure envelope:



Data generally lies outside the Tresca envelope.

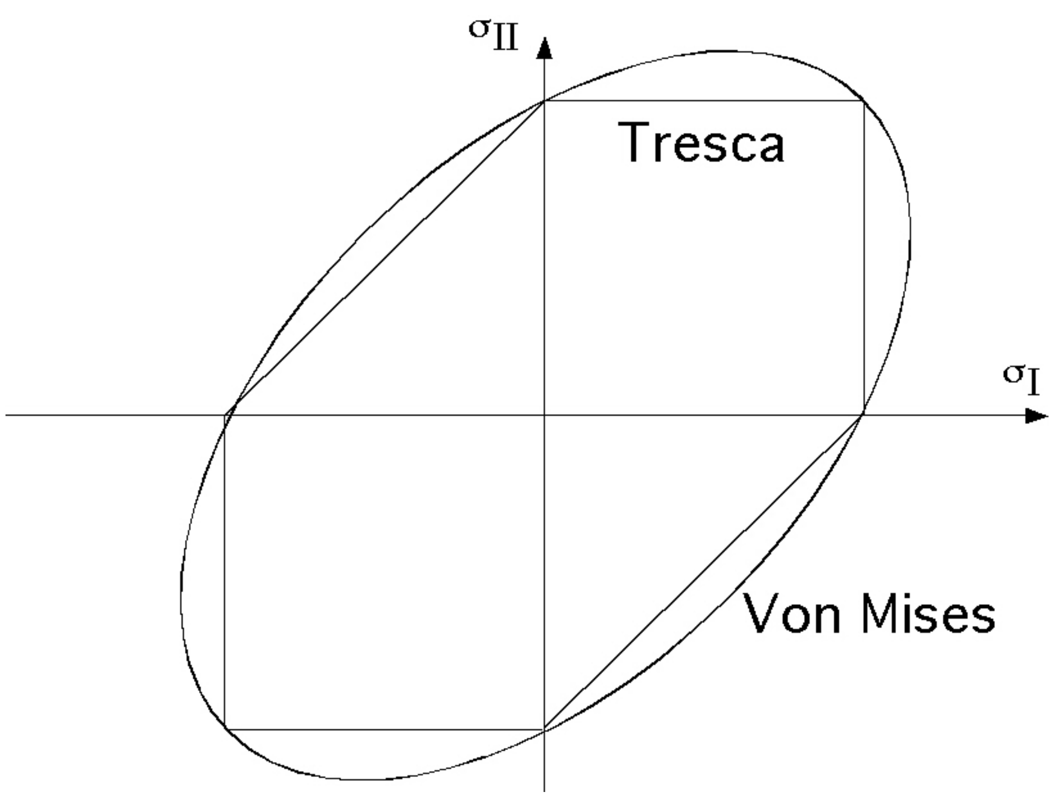
Propose second yield criterion:

2. Von Mises Yield Criterion.

Same basic idea, yield governed by shear, now include the three maximum shear stresses via:

$$(\sigma_I - \sigma_{II})^2 + (\sigma_{II} - \sigma_{III})^2 + (\sigma_{III} - \sigma_I)^2 \geq 2\sigma_y^2$$

Plotted for plane stress case ($\sigma_{III} = 0$) creates an elliptical yield surface.



Von Mises and Tresca criteria work quite well for metals.   Other failure criteria apply to other materials, ceramics, composites, polymers.

Useful to remember that they were developed well before there was any mechanistic understanding of dislocations and why shear stresses might govern yield.

M18 Fast Fracture and Toughness

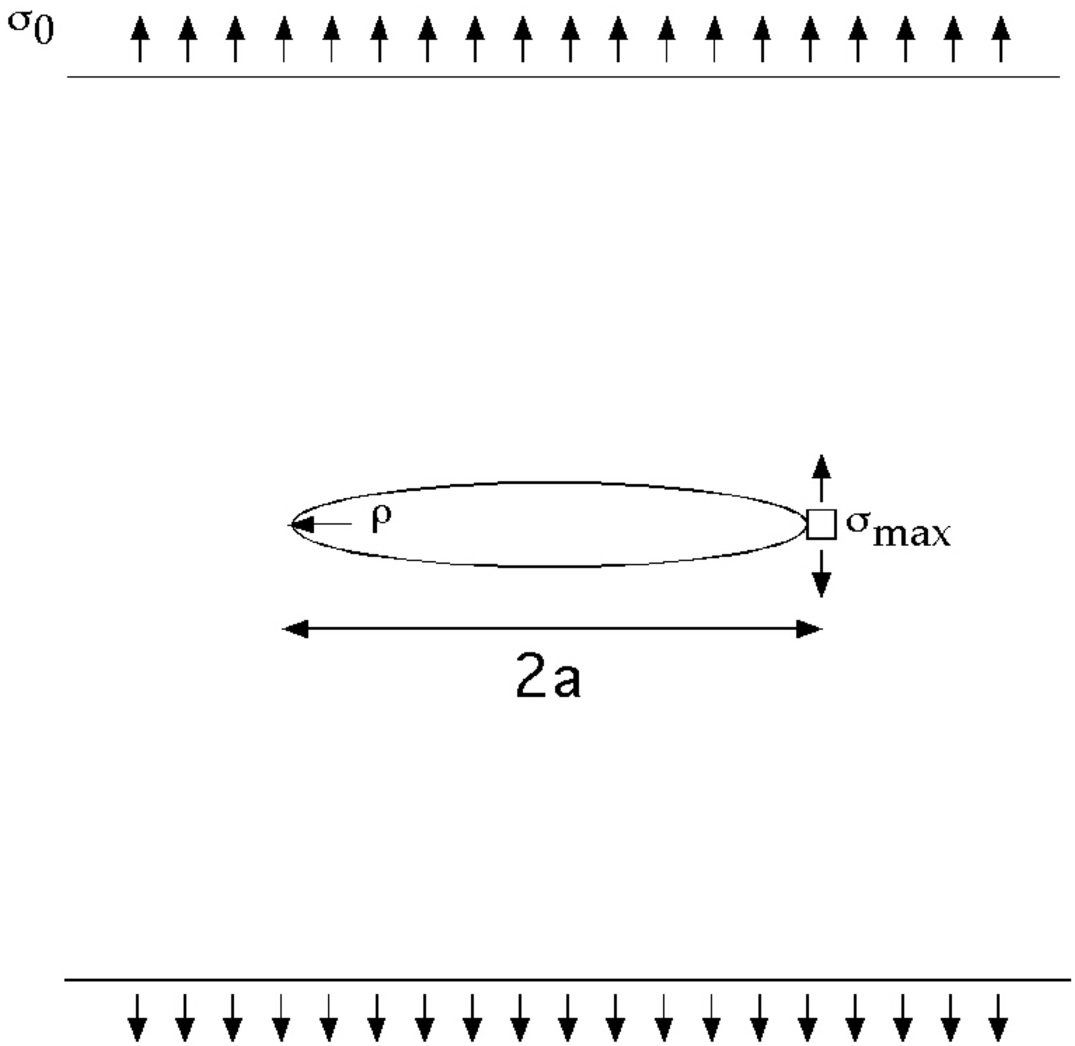
Reading: Ashby and Jones chapters. 13, 14

Yield and subsequent plasticity is a relative benign failure mode. It places a limit on the load carrying capability, but if that limit is exceeded, then the chief consequence is permanent deformation. This is obviously undesirable but will often not be catastrophic. In contrast, fast fracture, represents the sudden transition from an elastic response to catastrophic failure (rupture) via the propagation of a crack.

The objective of this lecture is to understand the mechanics of fracture and what controls the material property, toughness, that represents the resistance to fracture.

Basic mechanics concepts:

Stresses at an elliptical notch (see 16.20):



ρ is the radius of curvature of the tip of the notch, 2a is the length of the notch, and the local tensile stress at the tip is $\sigma_{\max}$ and the applied, remote stress is σ_0.

The stress concentration factor, K_T is given by:

$$K_T = \frac{\sigma_{\max}}{\sigma_0} = 1 + 2\sqrt{\frac{a}{\rho}}$$

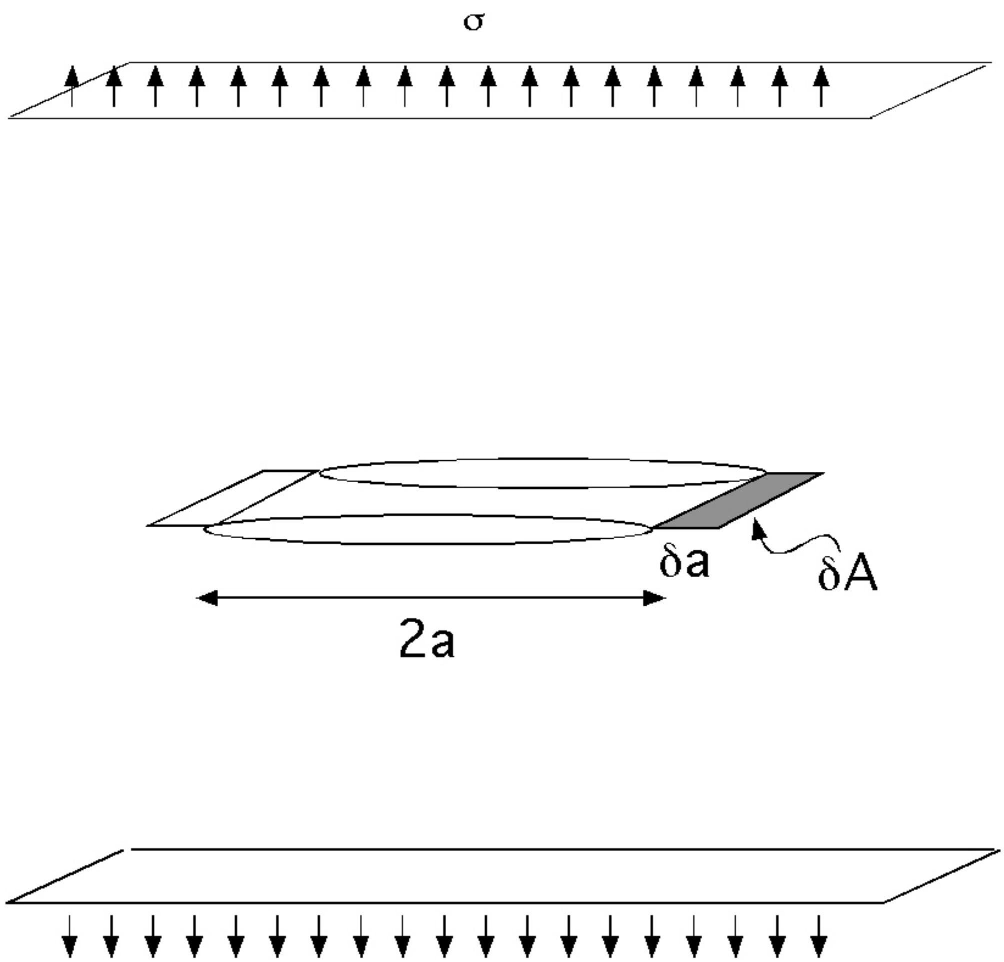
So as a/ρ increases the local stresses become higher. This is not sufficient to explain fracture, but it provides an explanation of why sharp cracks are detrimental.

The other key concept is that the propagation of a crack is controlled by the energy required to create new crack surfaces (and to do local deformation to the material).

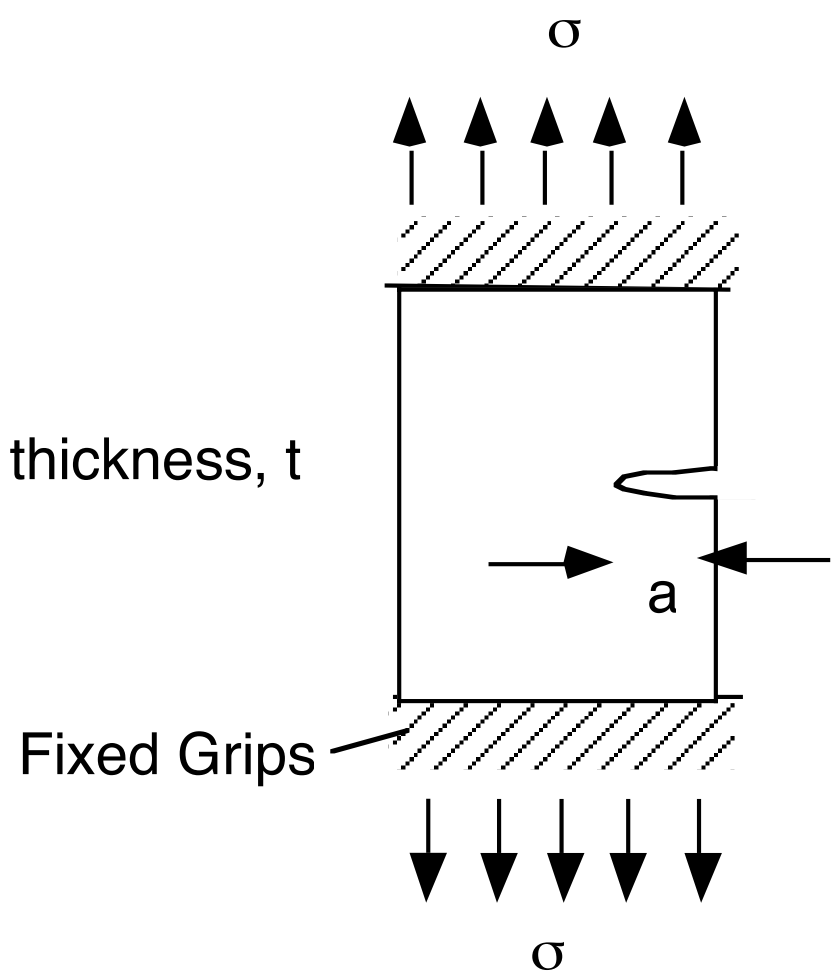
Griffith (1923) proposed that for fracture to occur the first law of thermodynamics must be satisfied:

$$\underbrace{\delta W_{External}}_{\text{Work by Loads}} - \underbrace{\delta U_{Internal}}_{\text{Elastic Strain Energy}} \geq G_c \underbrace{\delta A_{Area\ of}}_{\text{crack created}}$$

G_C is the energy required per unit are of crack surface. "Toughness" – units J/m^2



A key feature of fracture mechanics is that it predicts a dependence of the stress to propagate a crack on the size of the crack. This can be illustrated by considering a crack propagating in a plate of material under fixed grip conditions:



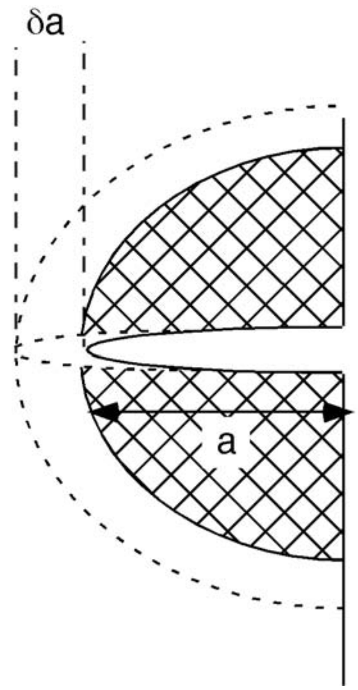
Displacements are fixed, so the loads cannot do work on the specimen (force x distance).

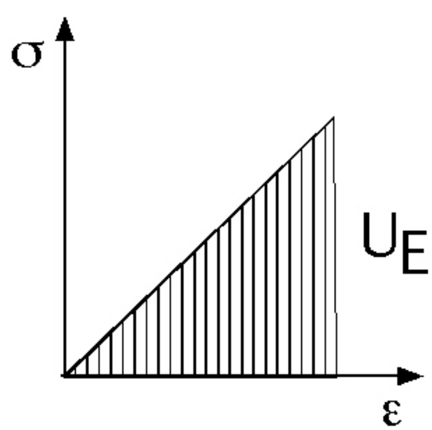
Hence:

$$-\delta U_{El} = G_c \delta A$$

Where does the elastic strain energy term come from, and how do we estimate it?

Consider material around crack: The crack surfaces are stress free. The edge of the specimen is also stress free. Assume that there is a semi-circular unloaded region around the crack. This grows with the crack





$$U_{El} = \frac{\sigma^2}{2E} \text{ per volume}$$

Hence elastic strain energy in material around crack is given by:

$$U_{El} \approx -\frac{\sigma^2}{2E}\frac{\pi a^2 t}{2}$$

And hence:

$$\delta U_{El} = \frac{dU_{El}}{da}\delta a = -\frac{\sigma^2}{2E}\frac{2\pi a t}{2}\delta a$$

And given that $t\delta a = \delta A$

Therefore $\dfrac{\pi a \sigma^2}{2E} \geq G_c$ for crack propagation.

It turns out that our estimate for the strain energy in the material around the crack is not particularly accurate, and is incorrect by a factor of 2.

i.e. $\dfrac{\pi a \sigma^2}{E} \geq G_c$ for crack propagation

The quantity $\frac{\pi a \sigma^2}{E}$ is often termed "the strain energy release rate", G so that fracture occurs when: $G \geq G_c$

or rearranging, to group the material properties, E, G_c, fracture occurs when:

$$\sigma\sqrt{\pi a} \geq EG_c$$

This turns out also to apply to the case of a cracked plate loaded with a constant load (vs. fixed grips).

For other geometries the same basic form is held and is often expressed as:

$$Y\sigma\sqrt{\pi a} \geq K_c$$

Where Y is a geometric factor, of order 1.

K_c is called the fracture toughness, and has units of MPa√m

$Y\sigma\sqrt{\pi a}$ is termed “the stress intensity factor, K. so the condition for fracture to occur can be expressed as:

$$K \geq K_c$$

Note that fracture depends both on the stress and the size of the crack in the structure.

Damage Tolerant Design:

We may wish to specify that our structure be able to tolerate a crack of a certain size (manufacturing tolerances, limitations in inspection, catastrophic events – uncontained fan blade loss). In which case, for a design stress, σ, the critical crack size is given by (if Y=1):

$$a_{crit} \geq \frac{1}{\pi}\left(\frac{K_c}{\sigma}\right)$$

Material Concepts

G_c and K_c are material properties. Can group according to classes of materials

Notice that tougher materials tend to have lower strengths.

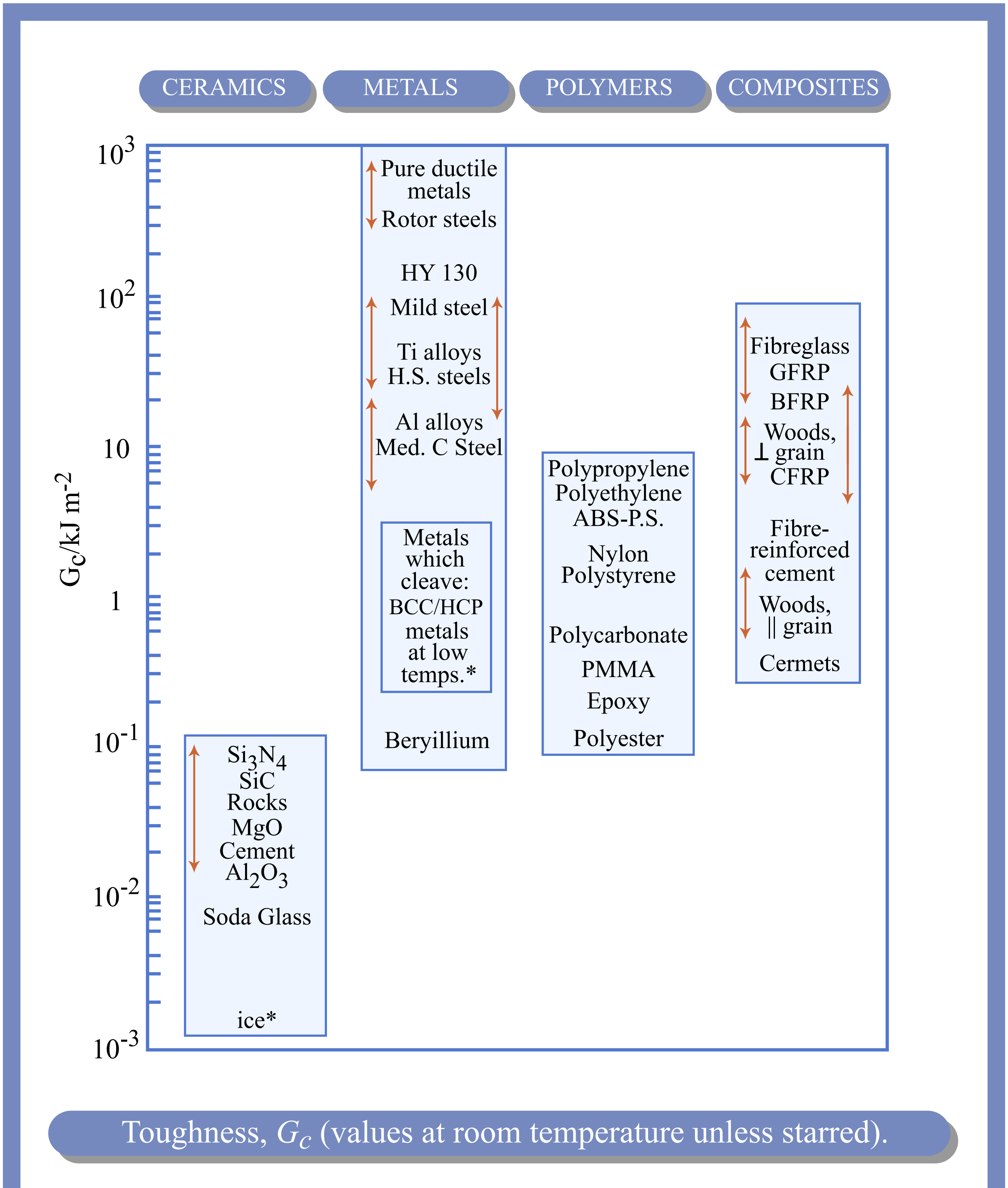
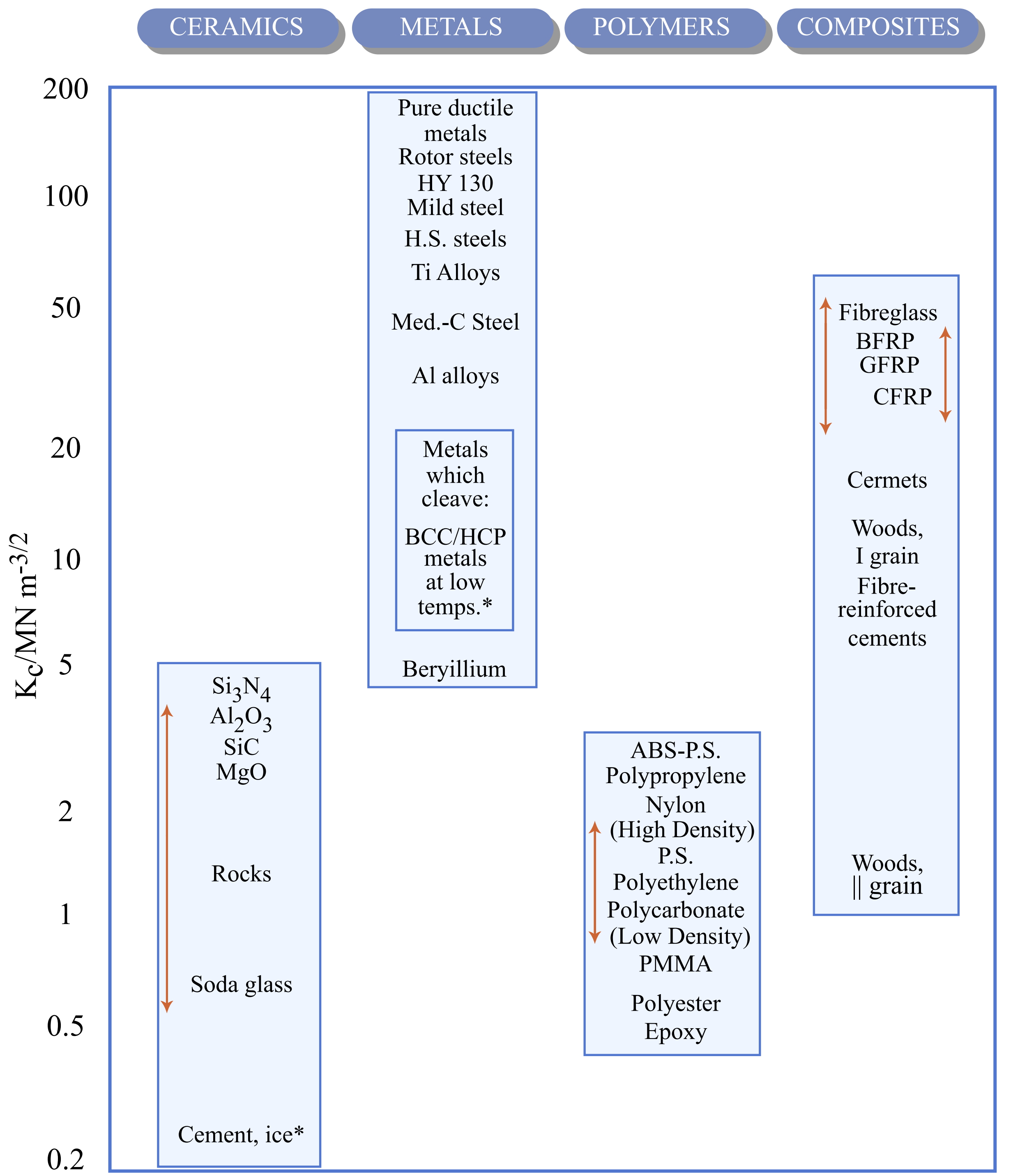
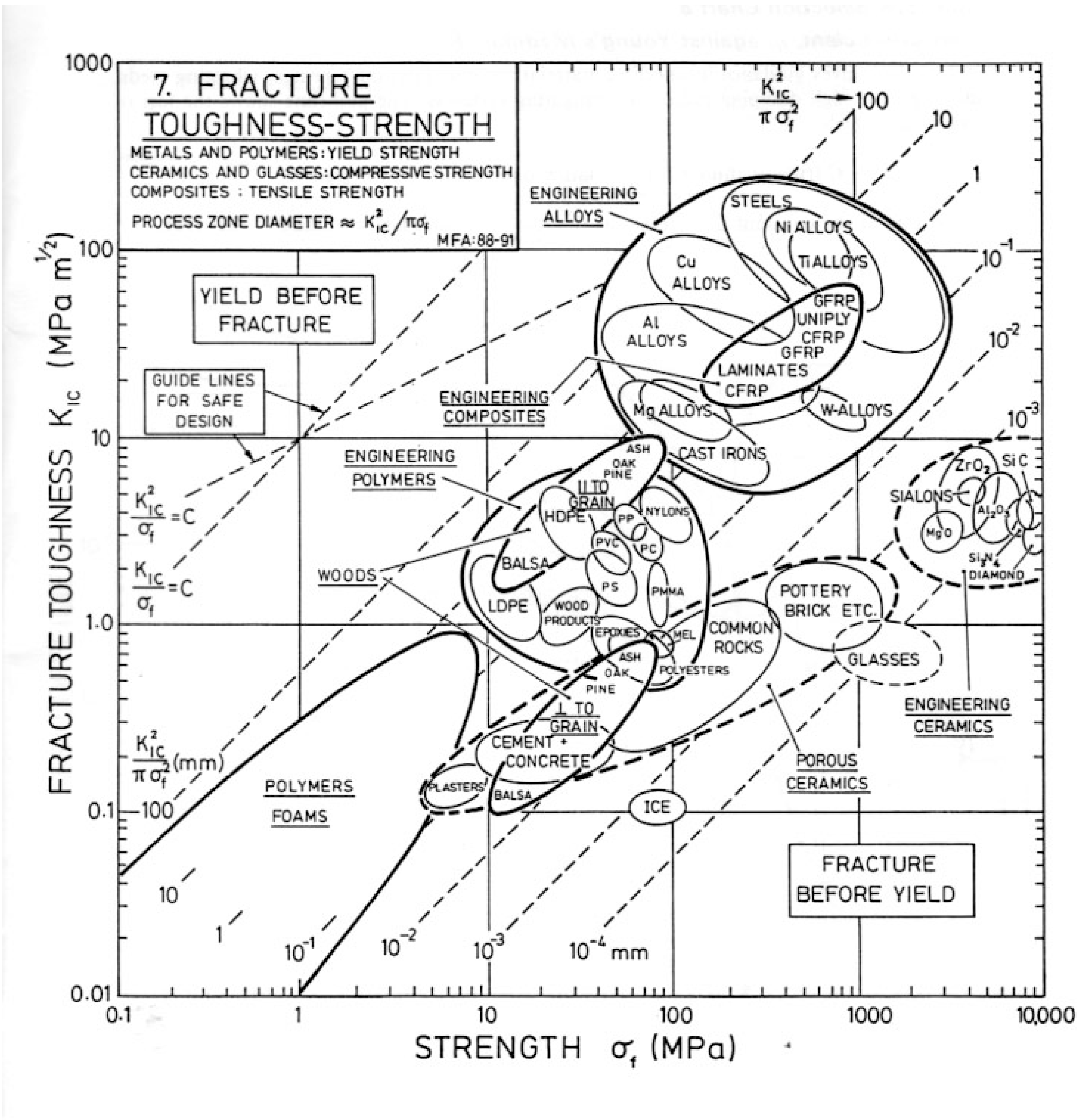


Image adapted from: Ashby, M. F., and D. R. H. Jones. *Engineering Materials 1*. Pergamon Press, 1980.



Fracture toughness, K_c (value at room temperature unless starred).

Image adapted from: Ashby, M. F., and D. R. H. Jones. *Engineering Materials 1*. Pergamon Press, 1980.



Ashby, Materials Selection

Micromechanisms of fast fracture – origins of toughness.

In a perfectly brittle material the fracture energy is controlled by the energy of the bonds between atoms at the crack tip:

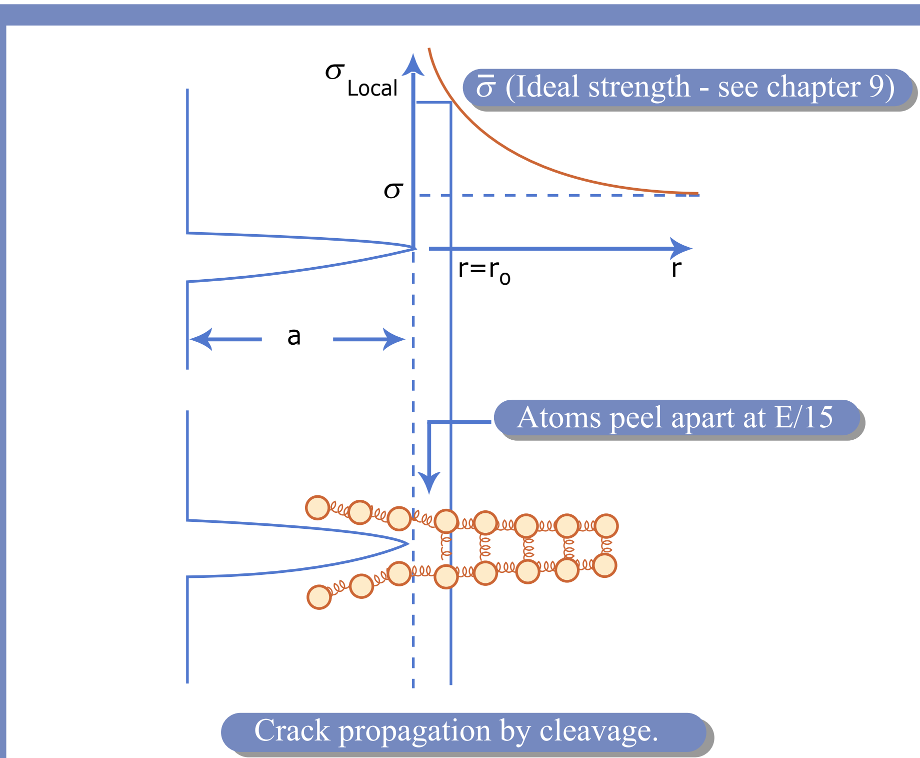


Image adapted from: Ashby, M. F., and D. R. H. Jones. *Engineering Materials 1*. Pergamon Press, 1980.

This applies to brittle materials such as ceramics and glasses. Toughness (G_c) values are in the range 0.1-10 J/m^2 and fracture toughness (K_c) ~ 1 MPa√m)

Metals tend to have much higher values of toughness and fracture toughness. Can understand why if we consider the stresses ahead of the tip of a sharp crack:

$$\sigma_{local} = \sigma + \sigma\sqrt{\frac{a}{2r}}$$

At some distance away from the crack tip the stresses will exceed the yield stress of the material, σ_y which will define a "plastic zone": $r_y = \dfrac{\sigma^2 a}{2\sigma_y^2}$

Which can be expressed in terms of the stress intensity factor, K:

$$r_y = \frac{K^2}{2\pi\sigma_y^2}$$

Therefore at the crack tip in a material with a low yield stress, there will be plastic deformation, which absorbs energy and blunts the crack tip:

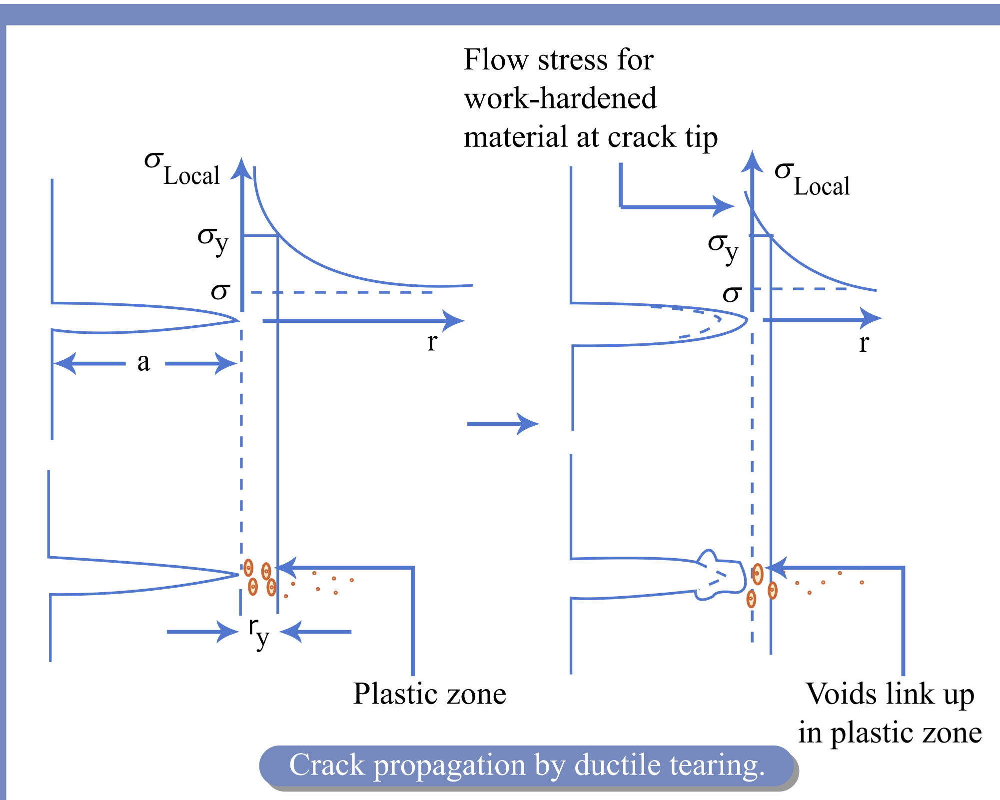


Image adapted from: Ashby, M. F., and D. R. H. Jones. *Engineering Materials 1*. Pergamon Press, 1980.

This is the main source of toughness in metals and explains why metal alloys with higher yield strengths will generally have lower values of toughness.

Next time: Fatigue.