Introduction to Computers and Programming

Prof. I. K. Lundqvist

Lecture 16
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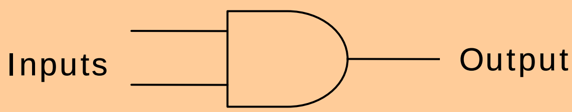
Today

- Boolean Logic
- Simplifying Formulae
- Constructing Logical Statements

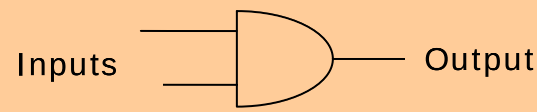
Pictorial Representation of AND and OR

AND



Inputs	Output
0 0	0
0 1	0
1 0	0
1 1	1

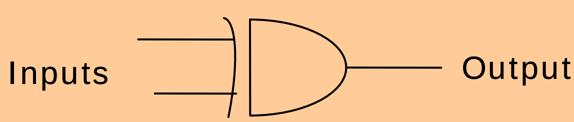
OR



Inputs	Output
0 0	0
0 1	1
1 0	1
1 1	1

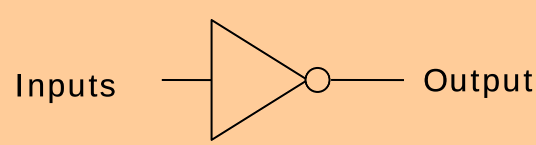
Pictorial Representation of XOR and NOT

XOR



Inputs	Output
0 0	0
0 1	1
1 0	1
1 1	0

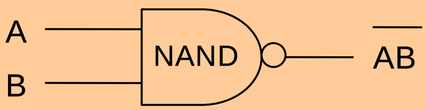
NOT



Inputs	Output
0	1
1	0

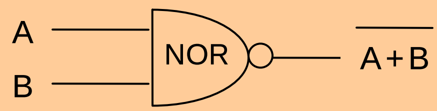
NAND and NOR

NAND



A	B	Out
0	0	1
0	1	1
1	0	1
1	1	0

NOR



A	B	Out
0	0	1
0	1	0
1	0	0
1	1	0

Proving DeMorgan's Theorem

$$(\overline{A} + \overline{B}) = \overline{A \bullet B}$$

$$(\overline{A} \bullet \overline{B}) = \overline{A + B}$$

A	B	$\overline{A}$	$\overline{B}$	$\overline{A}+\overline{B}$	$A \bullet B$	$\overline{A \bullet B}$
0	0	1	1	1	0	1
0	1	1	0	1	0	1
1	0	0	1	1	0	1
1	1	0	0	0	1	0

PSET ☺ / ☹

Truth Tables

A	B	C	F	
0	0	0	0	
0	0	1	1	$\overline{A}\cdot\overline{B}\cdot C$
0	1	0	1	$\overline{A}\cdot B\cdot\overline{C}$
0	1	1	1	$\overline{A}\cdot B\cdot C$
1	0	0	0	
1	0	1	1	$A\cdot\overline{B}\cdot C$
1	1	0	1	$A\cdot B\cdot\overline{C}$
1	1	1	0	

Condition that A is 0, B is 0, C is 1.

Function F is true if any of these and-terms are true!

OR

$$F = (\overline{A}\cdot\overline{B}\cdot C) + (\overline{A}\cdot B\cdot\overline{C}) + (\overline{A}\cdot B\cdot C) + (A\cdot\overline{B}\cdot C) + (A\cdot B\cdot\overline{C})$$

Sum-of-Products form

Minterms

- A minterm is a special product of literals, in which each input variable appears exactly once
- A function with **n** variables has $\mathbf{2^n}$ **minterms** (since each variable can appear complemented or not)
- A two-variable function, such as **f(x,y)**, has $\mathbf{2^2}$ = 4 **minterms**

Example-1

A	B	C	Output
0	0	0	0
0	0	1	**1**
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	**1**
1	1	0	**1**
1	1	1	**1**

→ 4 minterms

↓

$$\text{Output} = \overline{A}\bullet\overline{B}\bullet C + A\bullet\overline{B}\bullet C + A\bullet B\bullet\overline{C} + A\bullet B\bullet C$$

Boolean Algebra Theorems

ABC=(AB)C=A(BC), A+B+C=(A+B)+C=A+(B+C)	**AND, OR are associative**
AB=BA, A+B=B+A	**AND, OR operations are commutative**
A+BC=(A+B)(A+C), A(B+C)=AB+AC	**Forms of the distributive property**
$\overline{A+B}=\overline{A}\,\overline{B}$	**a form of DeMorgan's Theorem**
$\overline{AB}=\overline{A}+\overline{B}$	**a form of DeMorgan's Theorem**
$AA=A,\ A+A=A,\ \overline{A}+A=1,\ A\overline{A}=0,\ A=\overline{\overline{A}}$	**Single Variable Theorems**
$A+AB=A,\ A+\overline{A}B=A+B$	**Two-variable theorems**
$A1=A,\ A+1=1,\ A+0=A,\ A0=0,\ \overline{1}=0,\ \overline{0}=1$	**Identity and Null operations**

Expression Simplification
Sum-of-Products Minimization
(Example–1)

$$A \bullet B \bullet \overline{C} + A \bullet B \bullet C + A \bullet \overline{B} \bullet C + \overline{A} \bullet \overline{B} \bullet C$$

$\Rightarrow$ $A \bullet B \bullet (\overline{C} + C) + \overline{B} \bullet C \bullet (A + \overline{A})$ [distributive property]

$\Rightarrow$ $A \bullet B \bullet 1 + \overline{B} \bullet C \bullet (A + \overline{A})$ [single variable theorem]

$\Rightarrow$ $A \bullet B \bullet 1 + \overline{B} \bullet C \bullet 1$ [single variable theorem]

$\Rightarrow$ $A \bullet B + \overline{B} \bullet C \bullet 1$ [identity operation]

$\Rightarrow$ $A \bullet B + \overline{B} \bullet C$ [identity operation]

Karnaugh Maps
(Example-1)

Note: only 1 bit changes between adjacent cells

C \ AB	00	01	11	10
0	0	0	1	0
1	1	0	1	1

RED = $A \bullet B \bullet (\overline{C} + C) = A \bullet B$

GREEN = $C \bullet \overline{B} \bullet (\overline{A} + A) = C \bullet \overline{B}$

BLUE = $C \bullet (A \bullet B + A \bullet \overline{B}) = C \bullet A(B + \overline{B}) = C \bullet A$

Karnaugh Maps (Example-1)

Note: only 1 bit changes between adjacent cells

C \ AB	00	01	11	10
0	0	0	1	0
1	1	0	1	1

RED = $A \bullet B \bullet (\overline{C} + C) = A \bullet B$

GREEN = $C \bullet \overline{B} \bullet (\overline{A} + A) = C \bullet \overline{B}$

$\Rightarrow \quad A \bullet B + \overline{B} \bullet C$

2 Variable K-Maps

B \ A	0	1
0		
1		

B \ A	0	1
0	$\overline{A}\overline{B}$	$A\overline{B}$
1	$\overline{A}B$	AB

CQ 1

$\overline{A}\,\overline{B} + A\overline{B}$

I

B \ A	0	1
0	1	0
1	1	0

III

B \ A	0	1
0	1	1
1	0	0

II

B \ A	0	1
0	?	?
1	?	?

IV

B \ A	0	1
0	1	1
1	1	1

2 Variable K-Map Example

$\overline{A}\bullet\overline{B} + \overline{A}\bullet B$ →

B \ A	0	1
0	1	0
1	1	0

↓ $\overline{A}$

$\overline{A}\bullet\overline{B} + \overline{A}\bullet B$

$= \overline{A}(\overline{B} + B)$ [Distributive]

$= \overline{A} \bullet 1$ [$B + \overline{B} = 1$]

$= \overline{A}$ [$A \bullet 1 = A$]

Three Variable K-Map

C \ AB	00	01	11	10
0	1	0	1	0
1	1	0	0	1

$\overline{A}\bullet\overline{B}\bullet\overline{C} + \overline{A}\bullet\overline{B}\bullet C + A\bullet B\bullet\overline{C} + A\bullet\overline{B}\bullet C$

K-Map Rules of Simplification

CD \ AB	00	01	11	10
00	0	1	0	0
01	0	0	0	0
11	0	0	0	0
10	0	1	0	0

$\overline{A}\bullet B\bullet\overline{D}$

CD \ AB	00	01	11	10
00	1	1	0	0
01	1	1	0	0
11	1	1	0	0
10	1	1	0	0

$\overline{A}$

CD \ AB	00	01	11	10
00	1	0	0	1
01	0	0	0	0
11	0	0	0	0
10	1	0	0	1

$\overline{A}\bullet\overline{B}$

1. Circle the **largest groups possible**
2. Group dimensions must be a **power of 2**

Four Variable K-Maps
Example-2

Using a 4-variable K-Map, simplify the following Truth table

A	B	C	D	Output
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	**1**
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	**1**
1	0	1	0	0
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	**1**
1	1	1	1	0

CD \ AB	00	01	11	10
00	0	1	0	0
01	0	0	0	1
11	0	0	0	0
10	0	0	1	0

Output = $\overline{A}B\overline{C}\overline{D} + A\overline{B}\overline{C}D + ABC\overline{D}$

Four Variable K-Maps
Example-2

Using a 4-variable K-Map, simplify the following Truth table

A	B	C	D	Output
0	0	0	0	**1**
0	0	0	1	**1**
0	0	1	0	0
0	0	1	1	0
0	1	0	0	**1**
0	1	0	1	**1**
0	1	1	0	0
0	1	1	1	0
1	0	0	0	**1**
1	0	0	1	0
1	0	1	0	**1**
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	**1**
1	1	1	1	**1**

CD \ AB	00	01	11	10
00	1	1	0	1
01	1	1	0	0
11	0	0	1	0
10	0	0	1	1

Output = $\overline{A}\overline{C} + ABC + A\overline{B}\overline{D}$

Product-of-Sums from a Truth Table

A	B	C	F	$\overline{F}$
0	0	0	0	1
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	1	0
1	0	1	1	0
1	1	0	1	0
1	1	1	1	0

Find an expression for $\overline{F}$

$$\overline{F} = \overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}C + \overline{A}B\overline{C}$$

$$F = \overline{\overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}C + \overline{A}B\overline{C}}$$

$$F = \overline{\overline{A}\,\overline{B}\,\overline{C}} \bullet \overline{\overline{A}\,\overline{B}C} \bullet \overline{\overline{A}B\overline{C}}$$

$$F = (A+B+C) \bullet (A+B+\overline{C}) \bullet (A+\overline{B}+C)$$