**Massachusetts Institute of Technology
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16.03/16.04 Unified Engineering III, IV
Spring 2004

Problem Set 12

Name: _____

Due Date: 5/4/04

	Time Spent (min)
CP16	
P8	
P9	
S14	
S15	
Study Time	

Announcements: Q6C will be on Friday, April 30.

C16

The problems in this problem set cover lecture C16

1.

    a.   Using truth tables, show that $\overline{A} \cdot \overline{B} = \overline{(A + B)}$

    b.   Using K-Maps, simplify the following expression:

$$\overline{A} \cdot \overline{B} \cdot \overline{C} + \overline{A} \cdot \overline{B} \cdot C + A \cdot \overline{B} \cdot C + A \cdot \overline{B} \cdot \overline{C}$$

    c.   Using K-Maps, simplify the following expression:

$$A \cdot B \cdot D + \overline{B} \cdot C \cdot D + \overline{A} \cdot B \cdot C \cdot D + \overline{C} \cdot D$$

    d.   Simplify the same expression using the rules of simplification (Boolean Algebra Theorems).

Note: When using truth tables, list the complete table.
When using K-Maps, show the simplifications on the diagram.
When using rules of simplification, state the rule being used on the same line in square brackets.

2. Convert the following expression into product of sum form:

$$\overline{A} \cdot \overline{B} \cdot \overline{C} + \overline{A} \cdot B \cdot C + A \cdot B \cdot \overline{C} + A \cdot \overline{B} \cdot C$$

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Problem P8. (Propulsion) **(L.O. G)**

To answer this question, it will be necessary to make some estimates from the CFM56-003 engine.

Image removed due to copyright considerations.

Measure the inlet and exit blade angles near the root of the fan and the inlet blade angle of the first stator blade on the 3-stage low pressure compressor (the "booster" as labeled above).

a) Based on these measurements and assuming that the axial flow Mach number is 0.5, how fast does the low-speed/low pressure spool rotate? Please give your answer in RPM's (revolutions per minute). Make two estimates, one based on the flow angle into the fan and one based on the flow angle into the stator.

b) At this rotational speed, what is the relative Mach number at the tip of the fan blade?

[*Note: If you did not attend one of the lectures where we discussed the engine, the low-speed spool is the shaft that holds the fan blades and the first three stages of the compressor along with the low pressure turbine. The high-speed spool is on a concentric shaft outside of the low speed shaft. It holds the high pressure compressor and the high pressure turbine. You can see which parts are attached to the low speed spool by gently moving the fan back and forth.*]

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Problem P9. (Propulsion) **(L.O. G)**

Consider the turbine rotor shown below. The velocity (V_1) and absolute frame flow angle (β_1) entering the rotor are fixed. The flow angle leaving the rotor in the relative frame (β_2') is also fixed. Further, $\beta_1 = \beta_2'$. The rotor speed ($r\omega$) can be changed by varying the load on the shaft. Assume the axial velocity (w) entering and exiting the blade row is the same.

a) Draw velocity triangles in the relative and absolute frames for the different blade speeds shown below for both the inlet and exit to the rotor. (Note the inlet velocity components form a 3-4-5 triangle.)

rω = 0   rω = w   rω = 2w

INLET

EXIT
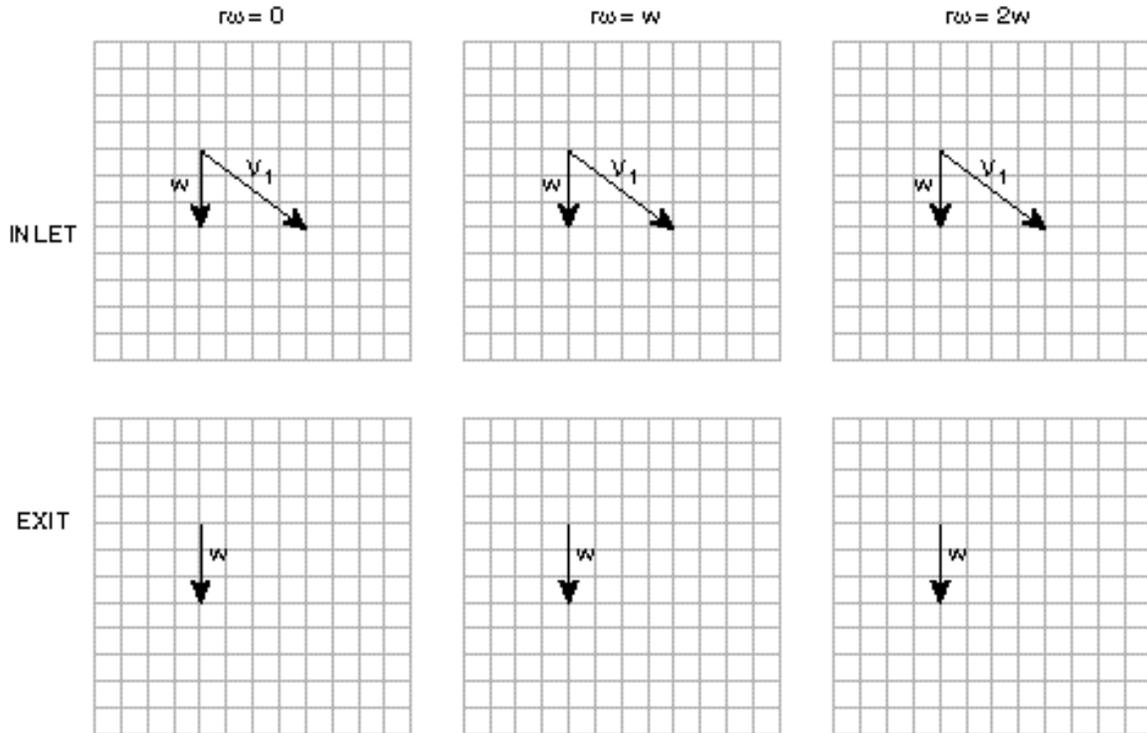
b)   For the three cases drawn in part a), which extracts the most power from the flow and why?

c) If you were asked to design for maximum power extraction and could choose any blade speed between 0 and rω = 2w, what speed would you choose?

You can argue this either graphically or with reference to the Euler Turbine Equation:

d)   For what rotational speed does the "turbine" in this example, begin to act like a compressor? What might you expect the aerodynamic performance to be for the blades in such a situation - good or bad, and why?

Problem S14 (Signals and Systems)

For each of the following Laplace transforms, find the inverse Laplace transform. Note that you can use the results of Problem S10 to help with the partial fraction expansions, but you will need to use the region of convergence to determine the inverse transform.

1. $G(s) = \dfrac{3s^2 + 3s - 10}{s^2 - 4}, \quad -2 < \mathrm{Re}[s] < 2$

2. $G(s) = \dfrac{3s^2 + 3s - 10}{s^2 - 4}, \quad \mathrm{Re}[s] < -2$

3. $G(s) = \dfrac{6s^2 + 26s + 26}{(s + 1)(s + 2)(s + 3)}, \quad -2 < \mathrm{Re}[s] < -1$

4. $G(s) = \dfrac{6s^2 + 26s + 26}{(s + 1)(s + 2)(s + 3)}, \quad -3 < \mathrm{Re}[s] < -2$

5. $G(s) = \dfrac{4s^2 + 11s + 9}{(s + 1)^2(s + 2)}, \quad -2 < \mathrm{Re}[s] < -1$

6. $G(s) = \dfrac{4s^2 + 11s + 9}{(s + 1)^2(s + 2)}, \quad \mathrm{Re}[s] < -2$

7. $G(s) = \dfrac{4s^3 + 11s^2 + 5s + 2}{s^2(s + 1)^2}, \quad -1 < \mathrm{Re}[s] < 0$

8. $G(s) = \dfrac{4s^3 + 11s^2 + 5s + 2}{s^2(s + 1)^2}, \quad \mathrm{Re}[s] < -1$

9. $G(s) = \dfrac{s^3 + 3s^2 + 9s + 12}{(s^2 + 4)(s^2 + 9)}, \quad \mathrm{Re}[s] < 0$

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Problem S15 (Signals and Systems)

Find the Fourier transforms of the following signals:

1.

$$g(t) = \delta(t - T)$$

   Note: The system with impulse response $g(t)$ produces an output that is the input delayed by T. Since delays occur frequently in signal processing, $G(j\omega)$ is an important transfer function.

2.

$$g(t) = \begin{cases} 1, & |t| \le T \\ 0, & |t| > T \end{cases}$$

   Note: Because $g(t)$ is symmetric, $G(j\omega)$ should be real. Please express your answer so that it is apparent that the answer is real.

3.

$$g(t) = \frac{1}{t^2 + T^2}$$

   Hint: If you find the integral hard to do, you might be able to find the answer using duality.

4.

$$g(t) = \frac{\sin \pi t/T}{\pi t/T}$$

   Hint: You almost certainly won't be able to do the FT integral directly. Use duality and the results of (2) above to find the answer. The $g(t)$ in this problem has important connections to, among other things, CD players!

5. Find the inverse transform of

$$G(j\omega) = \left(\frac{\sin \omega T}{\omega T} \right)^2$$

   using the results of part (2), and FT properties.