Problem S22 (Signals and Systems)

Consider a pulse similar to the Loran-C pulse, given by

$$h(t) = t^3 e^{-t/\tau}\sigma(t)\sin(2\pi f t) = g(t)w(t)$$

where

$$g(t) = te^{-t/\tau}\sigma(t)$$
$$w(t) = \sin(2\pi f t)$$

(a) Find the *centroid* of the pulse envelope, given by

$$\bar{t} = \frac{\int t g^2(t)\,dt}{\int g^2(t)\,dt}$$

(b) Find the duration of the envelope, given by

$$\Delta t = 2\left(\frac{\int (t-\bar{t})^2 g^2(t)\,dt}{\int g^2(t)\,dt}\right)^{\frac{1}{2}}$$

(c)

$$\Delta\omega = 2\left(\frac{\int \dot{g}^2(t)\,dt}{\int g^2(t)\,dt}\right)^{\frac{1}{2}}$$

(d) How does the duration-bandwidth product compare to the theoretical minimum?