1. From S13,

$$A = \begin{bmatrix} 0 & 1/L \\ -1/C & -1/RC \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ -1/RC \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 1 \end{bmatrix} \qquad D = \begin{bmatrix} 1 \end{bmatrix}$$

First find $(sI-A)^{-1}$:

$$sI - A = \begin{bmatrix} s & -1/L \\ 1/C & s + 1/RC \end{bmatrix}$$

For any matrix M, $M^{-1} = \dfrac{\operatorname{adj}(M)}{\det(M)}$. For 2×2 matrices, this gives:

$$(sI-A)^{-1} = \frac{1}{s^2 + \frac{s}{RC} + \frac{1}{LC}} \begin{bmatrix} s + 1/RC & +1/L \\ -1/C & s \end{bmatrix}$$

Now find $G(s) = C(sI-A)^{-1}B + D$:

$$C(sI-A)^{-1} =$$

$$\frac{1}{s^2 + \frac{s}{RC} + \frac{1}{LC}} \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} s + 1/RC & 1/L \\ -1/C & s \end{bmatrix}$$

$$= \frac{1}{s^2 + \frac{s}{RC} + \frac{1}{LC}} \begin{bmatrix} -1/C & s \end{bmatrix}$$

Then

$$C(sI-A)^{-1}B = \frac{1}{s^2+\frac{s}{RC}+\frac{1}{LC}}\begin{bmatrix} -1/C & s \end{bmatrix}\begin{bmatrix} 0 \\ -1/RC \end{bmatrix}$$

$$= \frac{-s/RC}{s^2+\frac{s}{RC}+\frac{1}{LC}}$$

Finally,

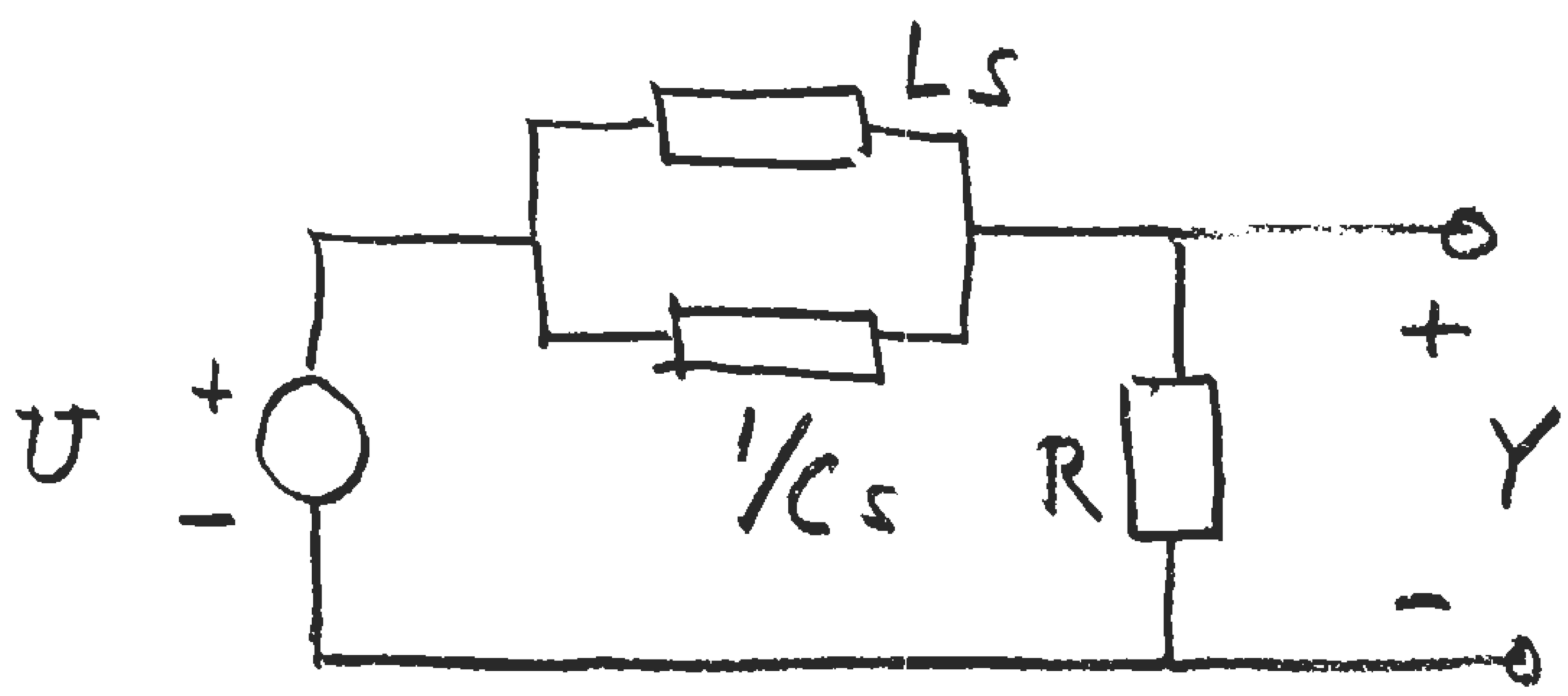
$$G(s) = C(sI-A)^{-1}B + D$$

$$= \frac{-s/RC}{s^2+s/RC+1/LC} + 1$$

$$= \frac{s^2+1/LC}{s^2+s/RC+1/LC}$$

$$\boxed{G(s) = \frac{s^2+1/LC}{s^2+s/RC+1/LC}}$$

2. We can also find $G(s)$ by impedance methods. Redraw the circuit:



The inductor and capacitor are in parallel. The combined impedance is

$$Ls \,\|\, \frac{1}{Cs} = \frac{(Ls)(1/Cs)}{Ls + 1/Cs}$$

$$= \frac{Ls}{LCs^2 + 1}$$

with this impedance, the circuit becomes a voltage divider:

$$Y = \frac{R}{R + \dfrac{Ls}{LCs^2+1}} \cdot v$$

$$= \frac{RLCs^2 + R}{RLCs^2 + R + Ls} v$$

$$= \underbrace{\frac{s^2 + 1/LC}{s^2 + \dfrac{s}{RC} + \dfrac{1}{LC}}}_{G(s)} v$$

So we get the same $G(s)$ as before.

3. For $L = 1H$, $C = 0.25F$, $R = 10\Omega$, the transfer function is

$$G(s) = \frac{s^2 + 4}{s^2 + 0.4s + 4}$$

For sinusoidal input, we can write

$$u(t) = \cos \omega t = \text{Real}\left[e^{j\omega t}\right]$$

Thus, $U = 1$

$s = j\omega$

So the ratio of output to input amplitudes is

$$|G(j\omega)| = \left|\frac{-\omega^2+4}{-\omega^2+0.4j\omega+4}\right|$$

This transfer function magnitude can be plotted by hand, or by using, say, Matlab. My Matlab code is below:

```
>> w = 0:.001:5;
>> G = (-w.^2+4)./(-w.^2+0.4j*w+4);
>> plot(w,abs(G))
>> axis([0 5 0 1.5]); ylabel('Magnitude of G(j\omega)'); xlabel('Frequency, \omega (rad/sec)');
>> print -depsc notch.eps
```

The resulting plot is on the next page. You can see why it is called a notch filter — the plot has a notch at the resonant frequency.

No. 5505
Engineer's Computation Pad