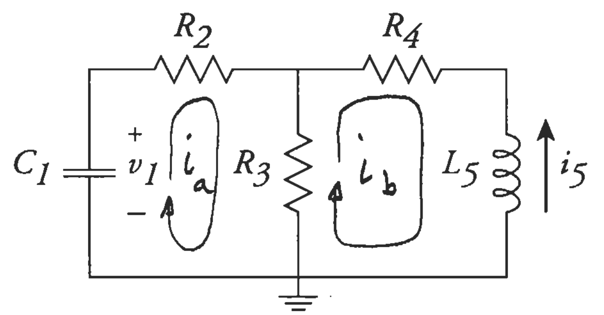
To solve, you can use the node method or loop method. It's easier with loop method. To solve, write KVL around 2 loops, plus capacitor constitutive law:



$$i_a: \quad (R_2 + R_3)\, i_a - R_3\, i_b - v_1 = 0$$

$$i_b: \quad -R_3\, i_a + \left(R_3 + R_4 + L_5 \frac{d}{dt}\right) i_b = 0$$

$$C_1: \quad i_a + C_1 \frac{dv_1}{dt} = 0$$

(Note that $i_a = -C_1\, dv_1/dt$, because $i_1 = -i_a$)

Plugging in numbers,

$$8 i_a - 4 i_b - v_1 = 0$$

$$-4 i_a + \left(2\frac{d}{dt} + 5\right) i_b = 0$$

$$i_a + 0.5 \frac{dv_1}{dt} = 0$$

If we assume that

$$i_a(t) = I_a e^{st}$$
$$i_b(t) = I_b e^{st}$$
$$v_1(t) = V_1 e^{st}$$

then the above equations become

$$
\begin{aligned}
8 I_a \quad & - 4 I_b & - V_1 &= 0 \\
-4 I_a + & (2s + 5) I_b & &= 0 \\
I_a \quad & & + 0.5s &= 0
\end{aligned}
$$

In matrix form,

$$
\underbrace{\begin{bmatrix} 8 & -4 & -1 \\ -4 & 2s+5 & 0 \\ 1 & 0 & 0.5s \end{bmatrix}}_{M(s)} \begin{bmatrix} I_a \\ I_b \\ V_1 \end{bmatrix} = 0
$$

For this equation to have a solution,

$$
\begin{aligned}
\det(M(s)) &= 0 \\
&= 8\left[(2s+5)(0.5s) - (0)(0)\right] \\
&\quad + 4\left[(-4)(0.5s) - (1)(0)\right] \\
&\quad - 1\left[(-4)(0) - (1)(2s+5)\right] \\
&= \left(4s(2s+5)\right) - 8s + 2s + 5 \\
&= 8s^2 + 14s + 5 = 0
\end{aligned}
$$

The roots are

$$s_1 = -1.25 \ \text{sec}^{-1}$$

$$s_2 = -0.5 \ \text{sec}^{-1}$$

Now find the characteristics vectors:

$s_1 = -1.25$:

$$M(s_1) = \begin{bmatrix} 8 & -4 & -1 \\ -4 & 2.5 & 0 \\ 1 & 0 & -0.625 \end{bmatrix}$$

$M(s_1)$ can be row-reduced to obtain

$$\begin{bmatrix} 1 & -1/2 & -1/8 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ V_1 \end{bmatrix} = \underline{0}$$

One solution is

$$\begin{bmatrix} I_a \\ I_b \\ V_1 \end{bmatrix} = \begin{bmatrix} 5/8 \\ 1 \\ 1 \end{bmatrix}$$

Similary, for $s_2 = -0.5$,

$$M(s_2) = \begin{bmatrix} 8 & -4 & -1 \\ 4 & 4 & 0 \\ -1 & 0 & -0.25 \end{bmatrix}$$

which can be row-reduced to obtain

$$\begin{bmatrix} 1 & -1/2 & -1/8 \\ 0 & 1 & -1/4 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ V_1 \end{bmatrix} = \underline{0}$$

A solution is

$$\begin{bmatrix} I_a \\ I_b \\ V_1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix}$$

The general solution is then

$$\begin{pmatrix} i_a(t) \\ i_b(t) \\ v_1(t) \end{pmatrix} = a \begin{pmatrix} 5/8 \\ 1 \\ 1 \end{pmatrix} e^{-1.25t} + b \begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix} e^{-0.5t}$$

The initial conditions are

$$v_1(0) = 2V = a + 4b$$

$$\Rightarrow a + 4b = 2$$

$$i_5(0) = 1A = -i_b(0) = -a - b$$

$$\Rightarrow -a - b = 1$$

In matrix form,

$$\begin{bmatrix} 1 & 4 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

The solution is

$$a = -2$$

$$b = 1$$

Therefore,

$$v_1(t) = a e^{-1.25t} + 4b e^{-0.5t}$$
$$= \left(-2e^{-1.25t} + 4e^{-0.5t}\right) \text{ volts}$$

$$i_5(t) = -i_6(t)$$
$$= -a e^{-1.25t} - b e^{-0.5t}$$
$$= \left(2e^{-1.25t} - e^{-0.5t}\right) \text{ amps}$$