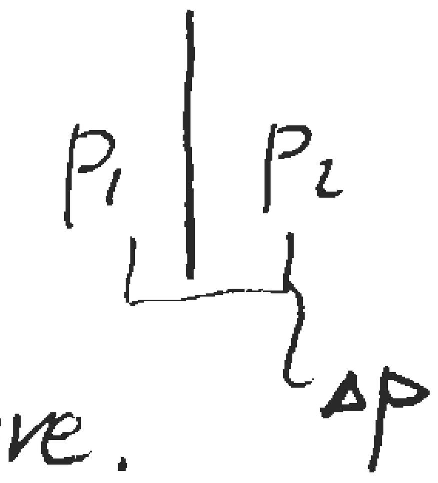
a) $dB = 20 \log_{10}\left(\frac{\Delta p}{20\times10^{-6}\,Pa}\right) = 120 \rightarrow \Delta p = 20\ Pa$



This Δp is the pressure change across a shock (sound) wave.

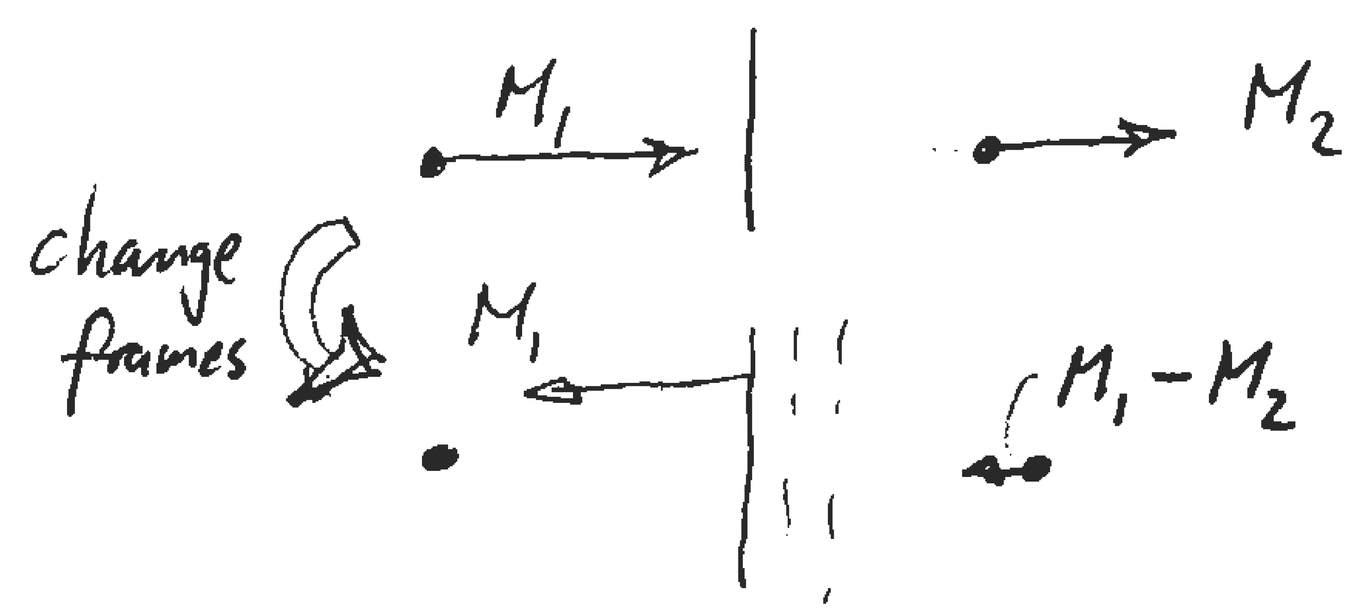
$\Delta p = p_2 - p_1$ where $p_1 \simeq 10^5\ Pa$ (atmosphere, sea level)

$$\frac{p_2}{p_1} = \frac{p_1 + \Delta p}{p_1} = 1 + \frac{\Delta p}{p_1} = 1 + \frac{20}{10000} = 1 + \frac{2\gamma}{\gamma+1}\left(M_1^2 - 1\right)$$

$$\Rightarrow \quad M_1^2 = 1.000171, \quad \boxed{M_1 = 1.000086} \quad \underline{\text{weak}}$$

This M_1 is the Mach number of the sound wave propagating into still air.

$u_1 \approx a_1$ (weak shock)



b) $$\frac{T_2}{T_1} = \left[1 + \frac{2\gamma}{\gamma+1}\left(M_1^2 - 1\right)\right]\frac{2 + (\gamma-1)M_1^2}{(\gamma+1)M_1^2} = 1.000057$$

For $T_1 = 300\,K^\circ$, $T_2 = 300.017^\circ$

$$\boxed{\Delta T = 0.017\,K^\circ}$$ pretty wimpy.

a) Anderson p. 502, problem 15.

Std Atmosphere

At 80000 ft = 15.15 mi = 24.38 km:

$$\left\{ \begin{aligned} a_1 &= 298 \text{ m/s} \\ \rho_1 &= 0.0437 \text{ kg/m}^3 \\ P_1 &= 5430 \text{ Pa} \\ T_1 &= 221 \text{ K}^\circ \end{aligned} \right.$$

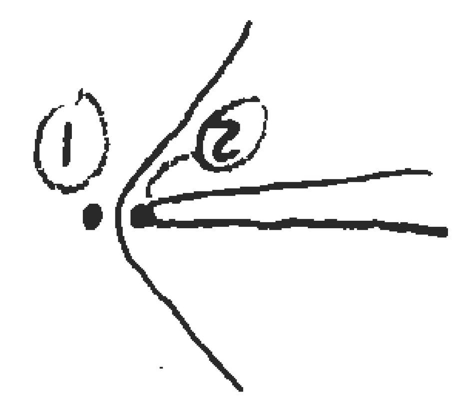
$$V_1 = V_\infty = 2112 \text{ mph} = 943.9 \text{ m/s}$$

$$M_1 = \frac{V_1}{a_1} = 3.17$$

$$\frac{T_2}{T_1} = \left[1 + \frac{2\gamma}{\gamma+1}(M_1^2 - 1)\right] \frac{2 + (\gamma-1)M_1^2}{(\gamma+1)M_1^2} = 2.885$$

$$\boxed{T_2 = T_1 \cdot 2.885 = 638 \text{ K}^\circ = 1148 \text{ R}^\circ = 656 \text{ F}^\circ}$$

b) P_{o2} will be behind bow shock at tip

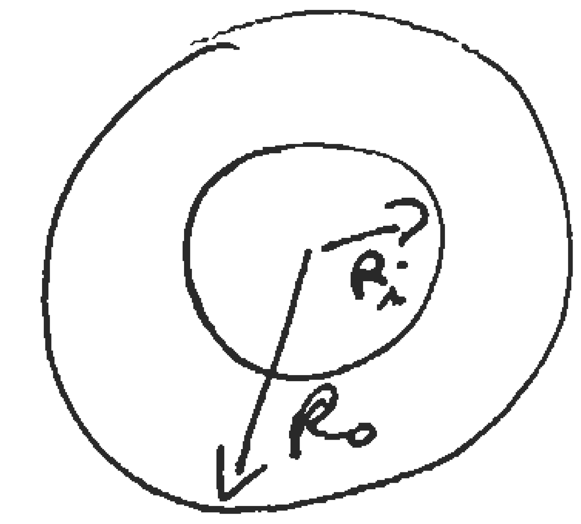


From Anderson Appendix B:

for $M_1 = 3.17 \rightarrow \frac{P_{o2}}{P_1} = 13.4$

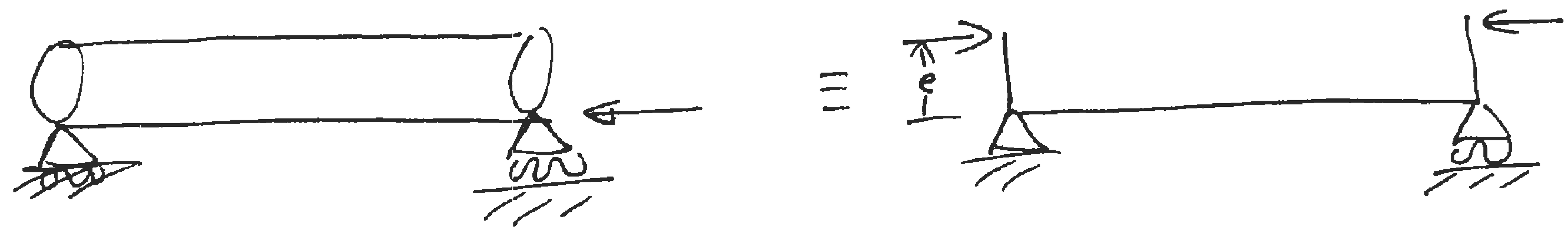
$$\boxed{P_{o2} = P_1 \cdot 13.4 = 7.28 \times 10^4 \text{ Pa}}$$

M14 $E = 70$ GPa



$R_i = \frac{4}{5} R_o$

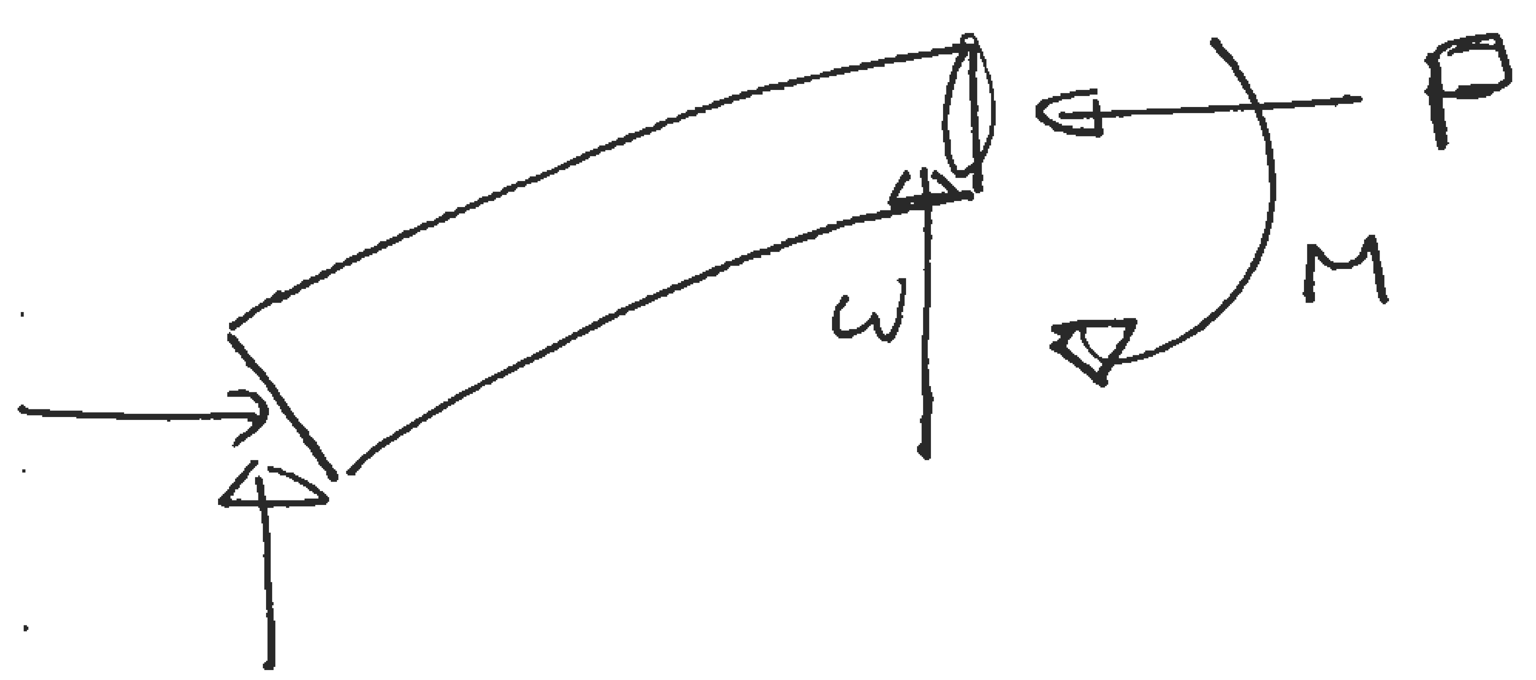
Basic Problem:



which we met in class

$$\therefore\ w = e\left(\frac{1-\cos\sqrt{\frac{P}{EI}}L}{\sin\sqrt{\frac{P}{EI}}L}\sin\sqrt{\frac{P}{EI}}x + \cos\sqrt{\frac{P}{EI}}x - 1\right)$$

The stress at a given point



$$\sigma = \underbrace{\frac{P}{A}}_{\text{axial}} \mp \underbrace{\frac{Mz}{I}}_{\text{bending}}$$

and $M = EI\dfrac{d^2w}{dx^2}$

$$\frac{d^2w}{dx^2} = -e\left(\frac{P}{EI}\right)\left[\frac{1-\cos\sqrt{\frac{P}{EI}}L}{\sin\sqrt{\frac{P}{EI}}L}\sin\sqrt{\frac{P}{EI}}x + \cos\sqrt{\frac{P}{EI}}x\right] -$$

moment will be a maximum at $x = \frac{L}{2}$ since $w = \max$.

$$M = EI\frac{d^2w}{dx^2}$$

$$M_{max} = -ep\left[\frac{1-\cos\sqrt{\frac{P}{EI}}L}{\sin\sqrt{\frac{P}{EI}}L}\sin\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right)+\cos\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right)\right]$$

let $\sqrt{\frac{P}{EI}}\frac{L}{2} = \theta$

$\sin 2\theta = 2\sin\theta\cos\theta$, $\cos 2\theta = \cos^2\theta - \sin^2\theta$

$$M_{max} = -ep\left[\frac{1-(\cos^2\theta - \sin^2\theta)}{2\cancel{\sin\theta}\cos\theta}\cancel{\sin\theta} + \cos\theta\right]$$

$1 = \cos^2\theta + \sin^2\theta$

$$M_{max} = -Pe\left[\frac{\cancel{\cos^2\theta}+\sin^2\theta-\cancel{\cos^2\theta}+\sin^2\theta}{2\cos\theta}+\cos\theta\right]$$

$$= -Pe\left[\frac{\sin^2\theta}{\cos\theta}\cancel{\tan\theta} + \cos\theta\right]$$

$$= -Pe\left[\frac{\sin^2\theta + \cos^2\theta}{\cos\theta}\right] = -Pe\left[\frac{1}{\cos\theta}\right] \; !$$

$$\therefore M_{max} = -Pe\left[\sec\sqrt{\frac{P}{EI}}\frac{L}{2}\right]$$

max for tensile stress

$$\text{Max stress} = \frac{P}{A} \mp \frac{MZ}{I}$$

Let $\frac{4}{5} = \alpha$

$R_i = \alpha R_o$

$A = \pi(R_o^2 - R_i^2) = \pi R_o^2(1-\alpha^2)$

$I = \frac{\pi}{4}(R_o^4 - R_i^4) = \frac{\pi R_o^4}{4}(1-\alpha^4)$

$Z = R_o$, $e = R_o$

$$\therefore\ \sigma = \frac{P}{\pi R_0^2(1-\alpha^2)} + \frac{P R_0}{\pi R_0^{\cancel{3}4}(1-\alpha^4)}\left[\sec\sqrt{\frac{P}{EI}}\frac{L}{2}\right]$$

$$\sigma = \frac{P}{\pi R_0^2}\left[\frac{1}{(1-\alpha^2)} + \frac{1}{R_0(1-\alpha^4)}\sec\sqrt{\frac{P}{EI}}\frac{L}{2}\right]$$

Need to iterate to solve. Calculate P_{crit} first $= 20\,KN$.

$$I = \frac{\pi \times (25\times10^{-3})^4}{4}\left(1-\left(\frac{4}{5}\right)^4\right) = 181.1\times10^{-9}$$

$EI = 12.7\times10^3$ $\quad \frac{L}{2} = 1.25\,m$ $\quad \frac{1}{25\times10^{-3}\left(1-\left(\frac{4}{5}\right)^4\right)} = 67.8$

$P = 10\,kN$. $\quad \sigma = 792\times10^6\,Pa$! too high $\quad \frac{1}{1-\alpha^2} = \frac{25}{9}$

$P = 1\,kN$ $\quad \sigma = 38.2\,MPa$

$\pi R_0^2 = 1.96\times10^{-3}$

$P = 3\,kN$ $\quad \sigma = 131\,MPa$

$P = 2\,kN$ $\quad \sigma = 81.5\,MPa$ $\quad \frac{1}{\sqrt{EI}}\frac{L}{2} = 11.10\times10^{-3}$

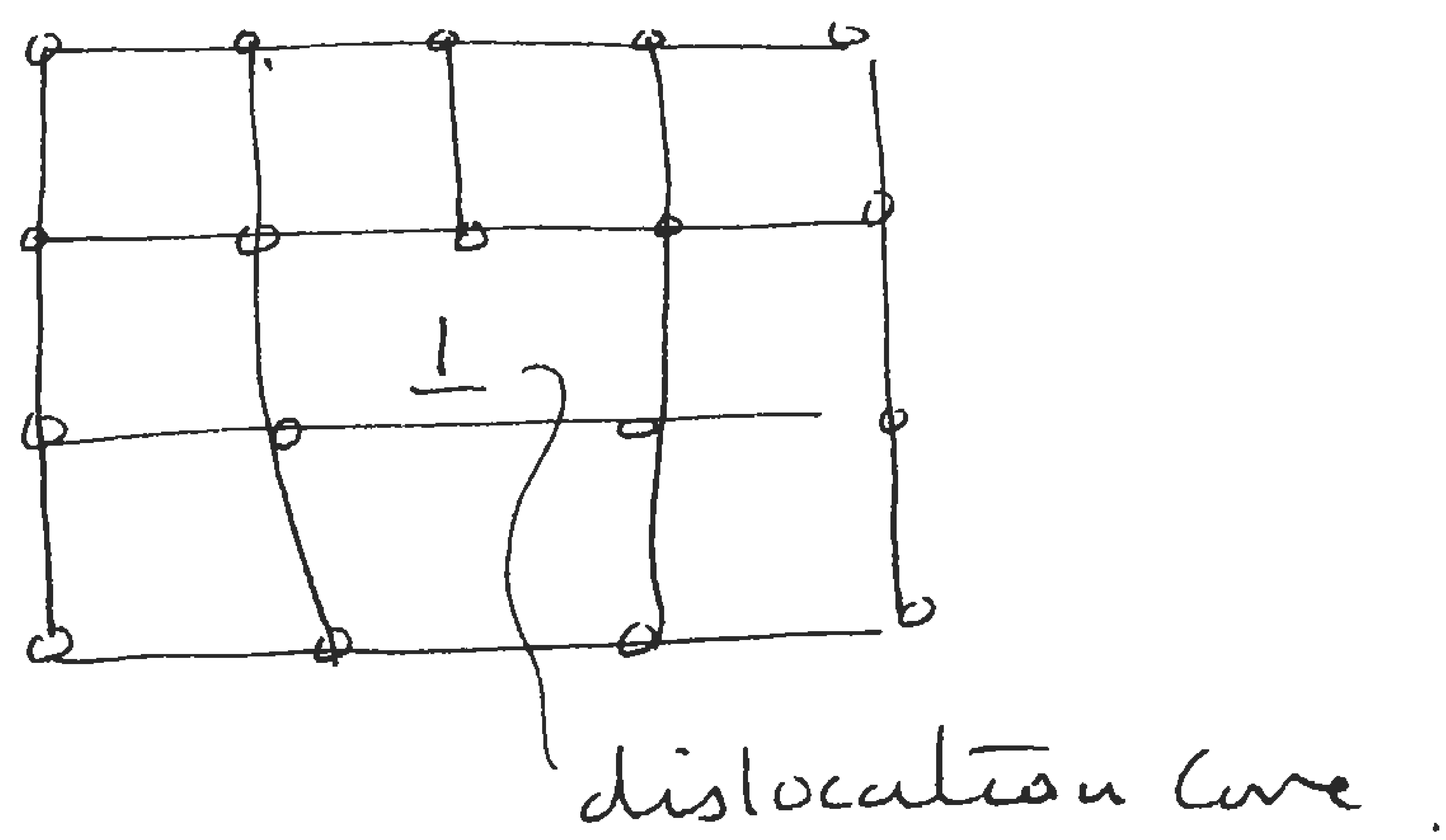
$\therefore$ max load $\simeq 2\,kN$

For perfect column $\quad P_{crit} = \frac{\pi^2 EI}{L^2}$

$$= \frac{\pi^2\times12.7\times10^3}{2.5^2} = 20\times10^3$$

M15

a) A dislocation is a defect in a crystal lattice consisting of an extra half-plane of atoms (edge dislocation).



Application of a shear stress allows the dislocation to move by breaking one row of atoms at a time.

b) Rolling allows the billet of metal to be reduced in thickness. The hot rolling allows large reductions in thickness by allowing creep and diffusion processes to occur. The final step of cold rolling allows work hardening to occur which increases the strength of the resulting material.

MIS

c) Polycrystalline material contains grain boundaries which increase the resistance to dislocation motion. There are no such boundaries in a single crystal.

d) The toughness of engineering alloys to a large extent reflects the contribution of plasticity to energy absorption at the crack tip. Lower yield stress materials tend to have higher toughnesses as they have more plastically deforming material at the crack tip.

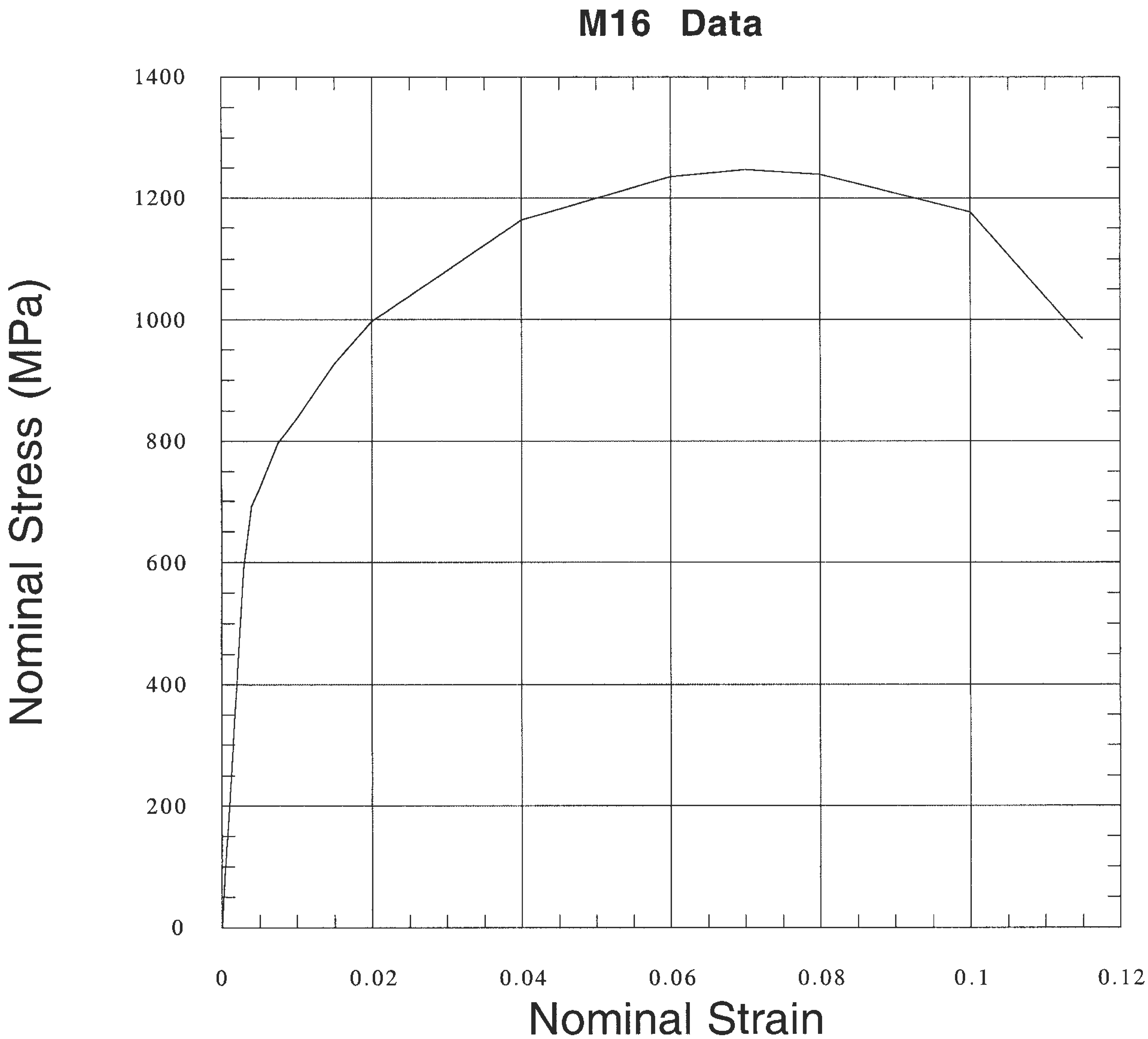
e) Carbon and glass are brittle materials. ~~Bg~~ Their strength is determined by the size of flaws (cracks) present. By drawing the fibers down to a small diameter the maximum flaw size is limited and a high strength results.

M15

f) Duralumin is an Al(Cu) alloy which is hardened by $CuAl_2$ precipitates. The extent to hardening is proportional to $\propto \frac{1}{L}$ the spacing of the precipitate particles.

The time dependence of the hardness reflects the growth of the particles from the solid solution. At short times the particles are too small to be effective at pinning dislocations. At very long times the particles have grown so large that "L" is also large. Thus there is a maximum at intermediate times when the particles are large enough to be effective and are still closely spaced.

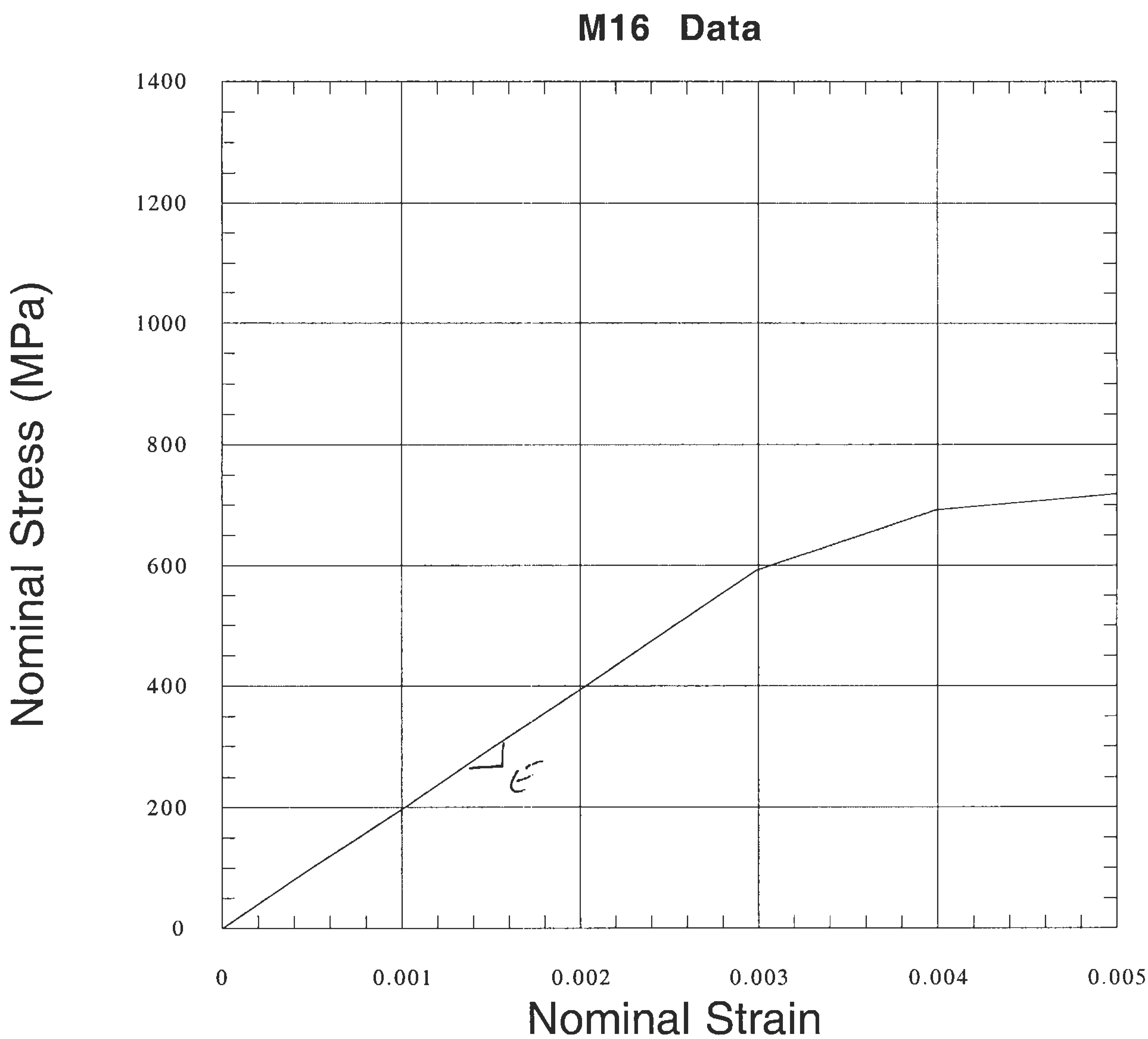
M16 a)



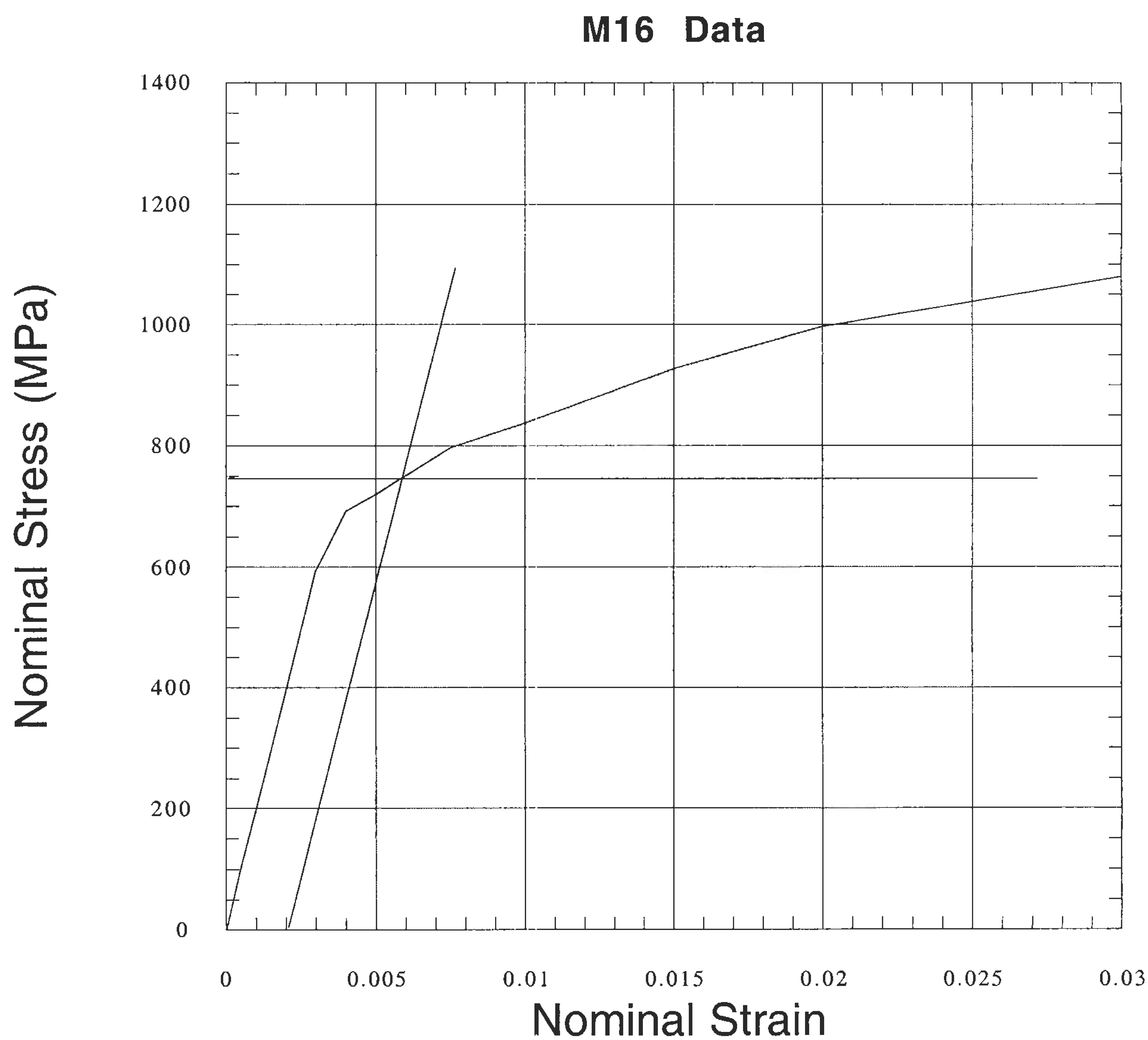
M16 Data
1400
1200
1000
800
600
400
200
0
Nominal Stress (MPa)
0
0.02
0.04
0.06
0.08
0.1
0.12
Nominal Strain

M16 b)

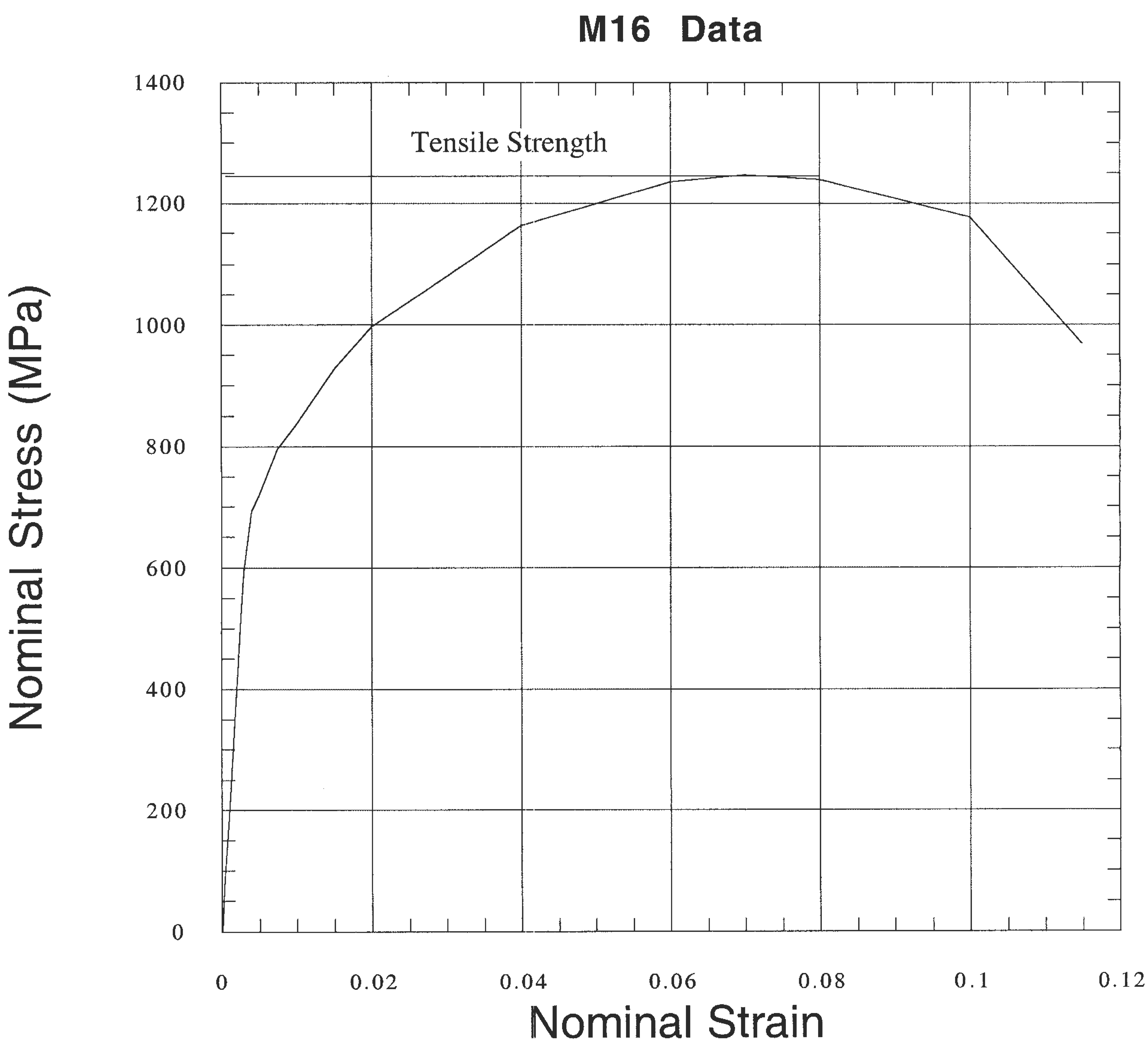
$$E = \frac{\sigma}{\varepsilon} = \frac{400 \times 10^6}{0.002} = 200\ \text{GPa}.$$



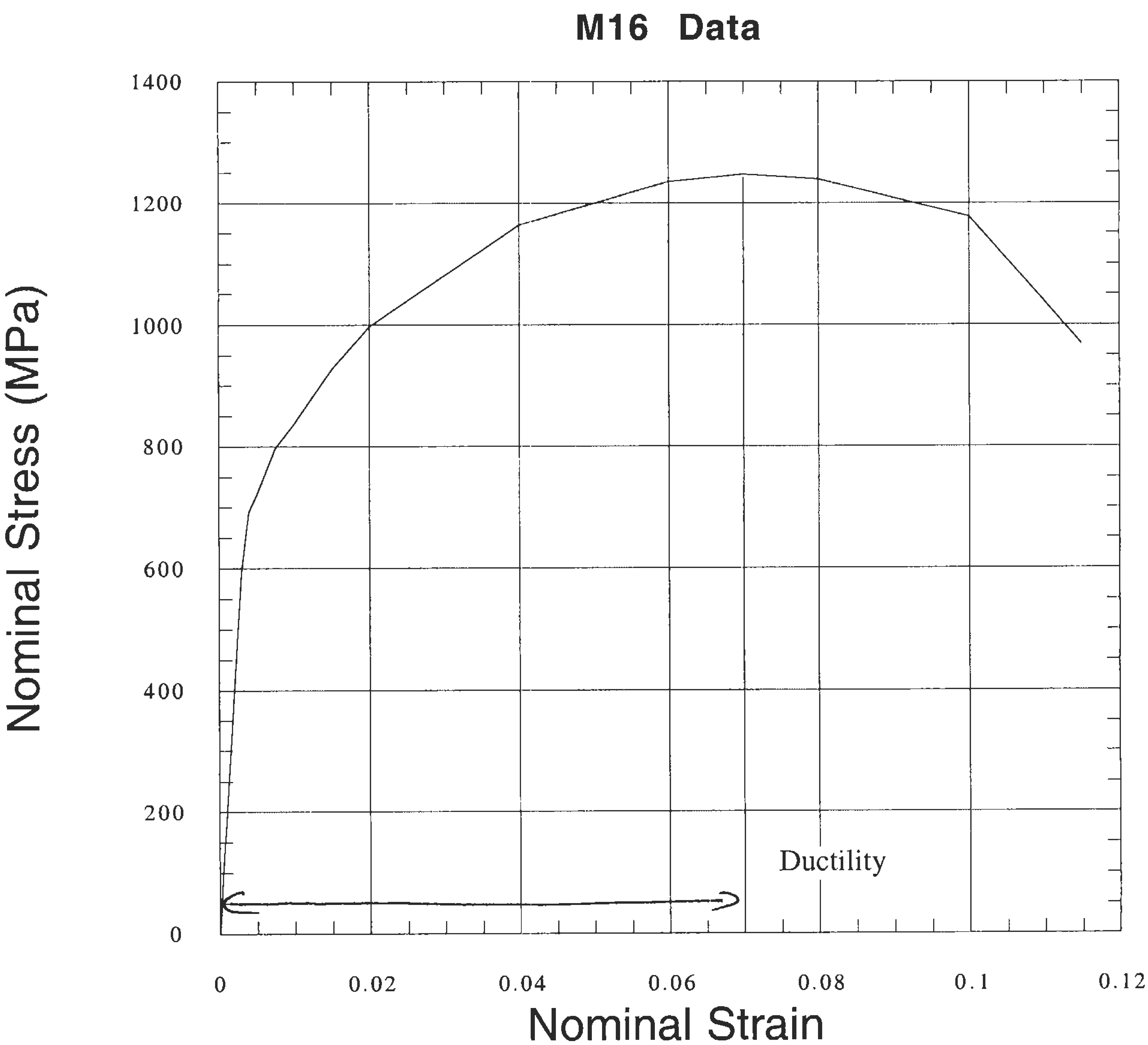
M16 c) 0.2% offset σ_y = 750 MPa.



M16 d) Tensile strength = 1250 MPa. ⇐



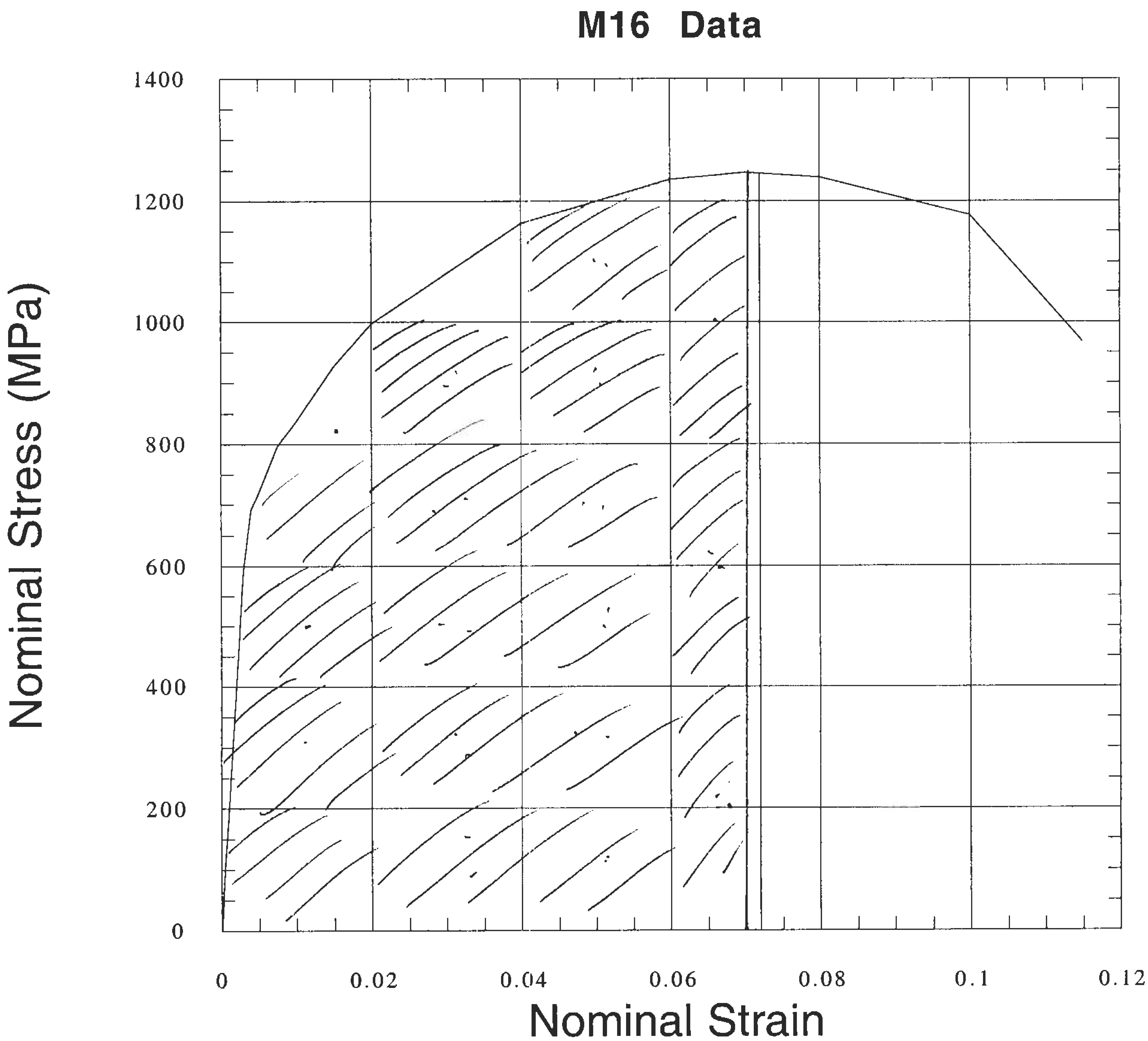
M16 e) Ductility = 0.07 = 7%



M16 f) $18\frac{1}{2}$ Squares 0.02×200 MPa $= 4$ MJ/m³

$\therefore$ Total energy $= 4 \times 10^6 \times 50.8 \times 10^{-3} \times \pi \times (6.4 \times 10^{-3})^2$

$= 26.4$ J.

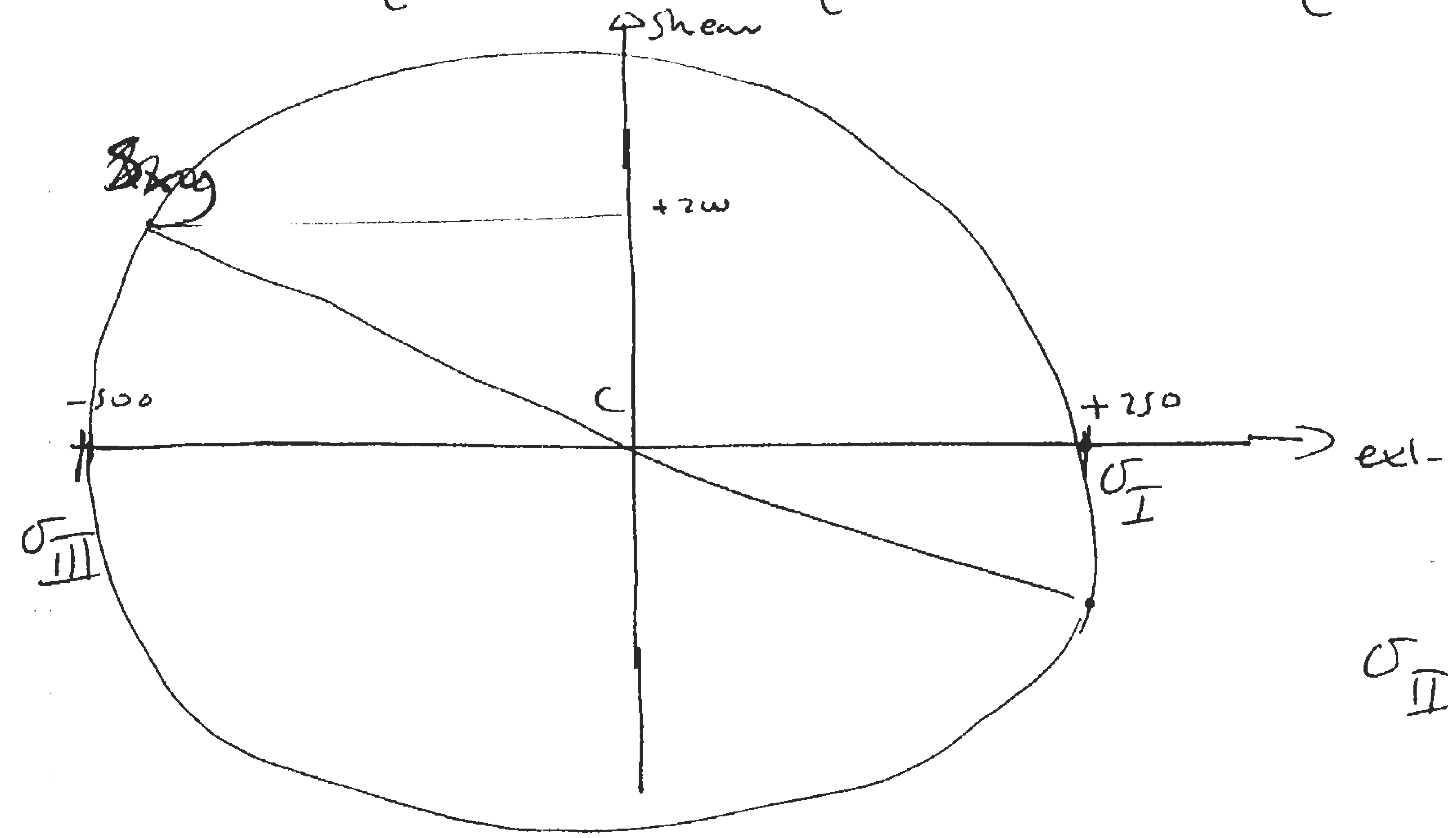


Approx $18\frac{1}{2}$ 0.02×200 MPa squares

M17

$$\sigma_y = \frac{N_y}{t} \qquad \sigma_x = \frac{N_x}{t} \qquad \sigma_{xy} = \frac{S_{xy}}{t}$$

①



$\sigma_{II} = 0$
through thickness

$$C = \frac{+250 - -500}{2} + -500 = -125$$

$$R = \sqrt{(250 - -125)^2 + (200)^2} = 425.$$

$$\therefore \quad \sigma_I = -125 + 425 = +\frac{300}{t} \times 10^3 \text{ Pa}$$

$$\sigma_{III} = -125 - 425 = -\frac{550}{t} \times 10^3 \text{ MPa}$$

Von Mises

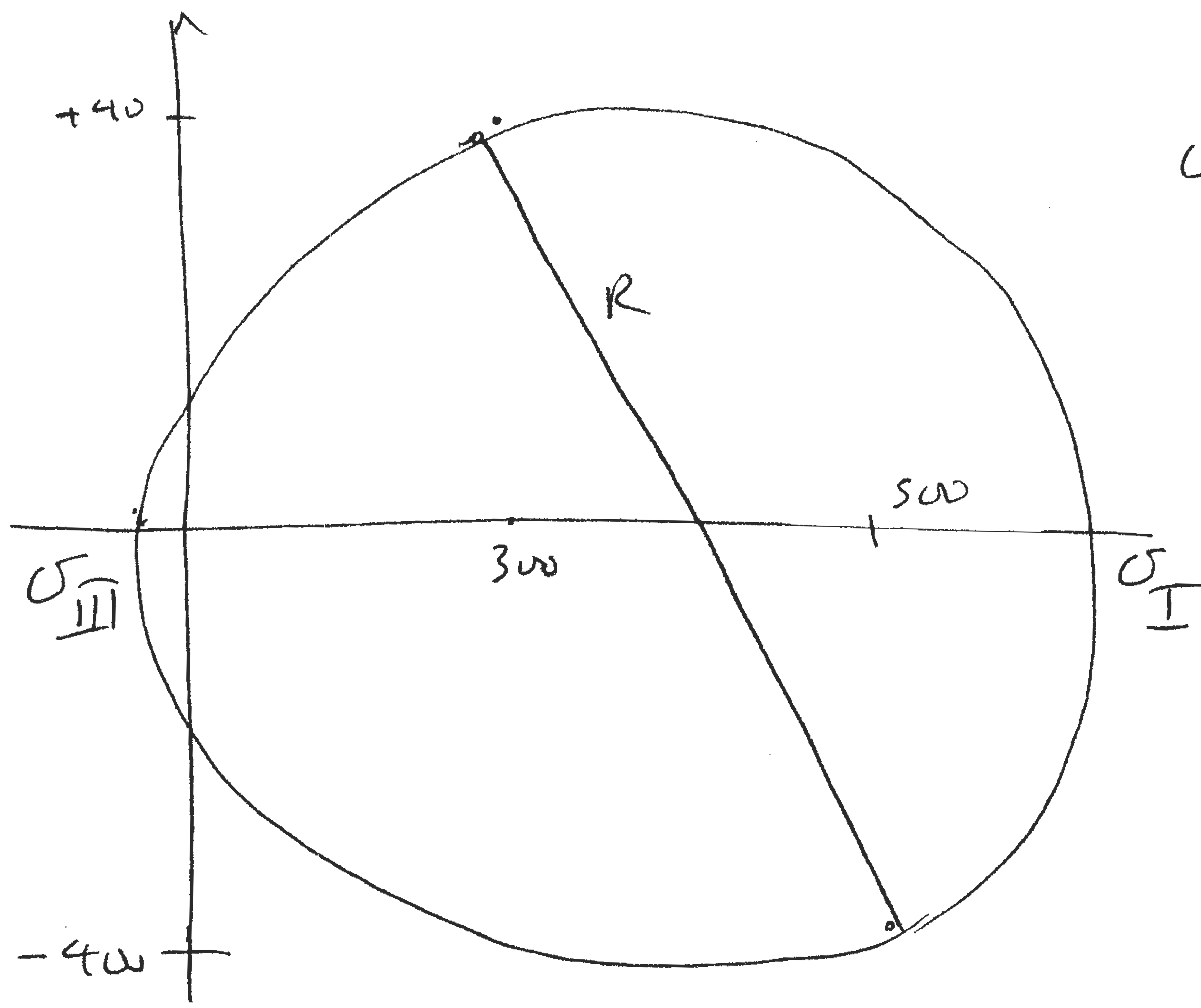
$$(\sigma_I - \sigma_{II})^2 + (\sigma_{II} - \sigma_{III})^2 + (\sigma_{III} - \sigma_I)^2 \geq 2\sigma_y^2$$

$$(300 - 0)^2 + (0 + 550)^2 + (-550 - 300)^2 = \frac{2 \times (500 \times 10^6)^2 t^2}{(10^3)^2 \times (1.5)^2}$$

safety factor

$$t = \sqrt{\frac{1115 \times 10^3 \times (10^3)^2 \times (1.5)^2}{2 \times (500 \times 10^6)^2}} = 2.24 \text{ mm}$$

(2)



$\sigma_{II} = 0$

$C = 400$.

$$R = \sqrt{(500-400)^2 + 400^2} = 412.3$$

$$\therefore \sigma_I = 400 + 412.3 = \frac{812.3}{t}$$

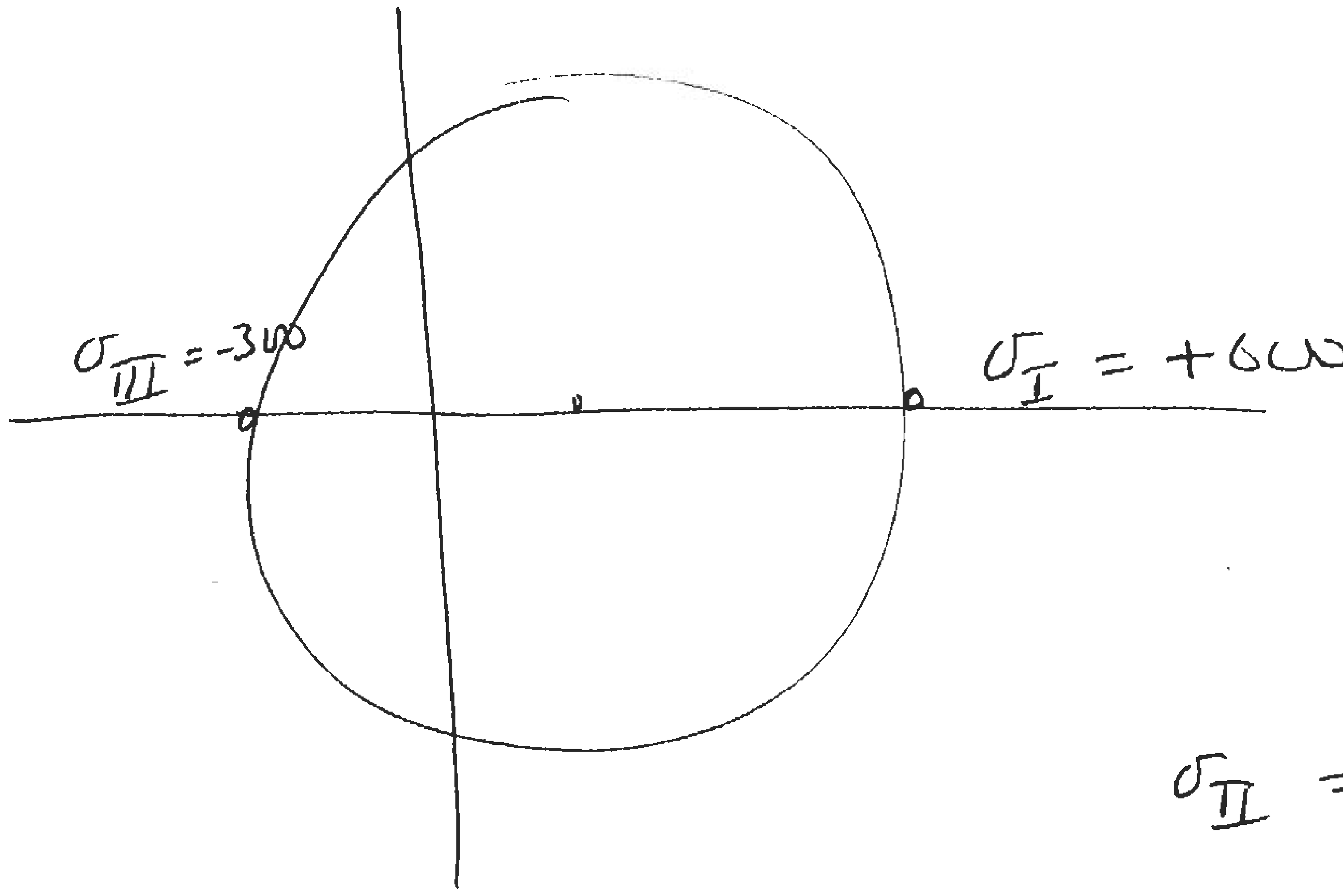
$$\sigma_{II} = 400 - 412.3 = \frac{-12.3}{t}$$

Von Mises $(812-0)^2 + (0+12.3)^2 + (812+12.3)^2 = 2 \times 500 \times 10^6$

$$t = \sqrt{\frac{1.34 \times 10^6 \times (10^3)^2 \times (1.5)^2}{2 \times (500 \times 10^6)^2}} = 2.45 \times 10^{-3}$$

$= 2.45$ mm

(3)



$\sigma_{II} = 0$

Von Mises $\quad (600)^2 + (-300)^2 + (600+300)^2 = \dfrac{2 \times (500 \times 10^6)^2 t^2}{(10^3)^2 \times (1.5)^2}$

$$t = \sqrt{\frac{1.26 \times 10^6 \times (10^3)^2 \times (1.5)^2}{2 \times (500 \times 10^6)^2}} = 2.38 \text{ mm.}$$

choose thickest required size $\quad$ 2.45 mm .

M18. Crack will propagate at highest ~~stress~~ tensile stress

This is in case (2) $\sigma_I = \dfrac{812.3\times10^{3}}{2.45\times10^{-3}} = 331.6$ MPa.

$\cancel{\sigma_f}$ = critical crack size $= \dfrac{1}{\pi}\left(\dfrac{K_{IC}}{\sigma}\right)^2 = 1.7\times10^{-3}$

$\therefore$ total crack size = ~~1.7~~ 3.4 mm $\Leftarrow$.

Need to reduce stress to point where critical crack half length = 2.5 mm $\Leftarrow$.

$\therefore \quad \sigma = 24\times10^{6} / \sqrt{\pi\times2.5\times10^{-3}} = 270.8$ MPa

$\therefore \quad \dfrac{812.3\times10^{3}}{t} = 270.8\text{ MPa} \Rightarrow t = \dfrac{812.3\times10^{3}}{270.8\times10^{6}} = 3\text{ mm} \Leftarrow$