Problem M22

A cantilever beam is to be made with a length L and circular cross section, radius R (to be specified by the designer), to support an applied moment, M.

a) What combination of material properties must be minimized to obtain the highest bending stiffness (i.e. minimum deflection for a given applied moment) beam for a given mass of material? (show your working). The deflection, δ, of a cantilever beam of this shape is given by:

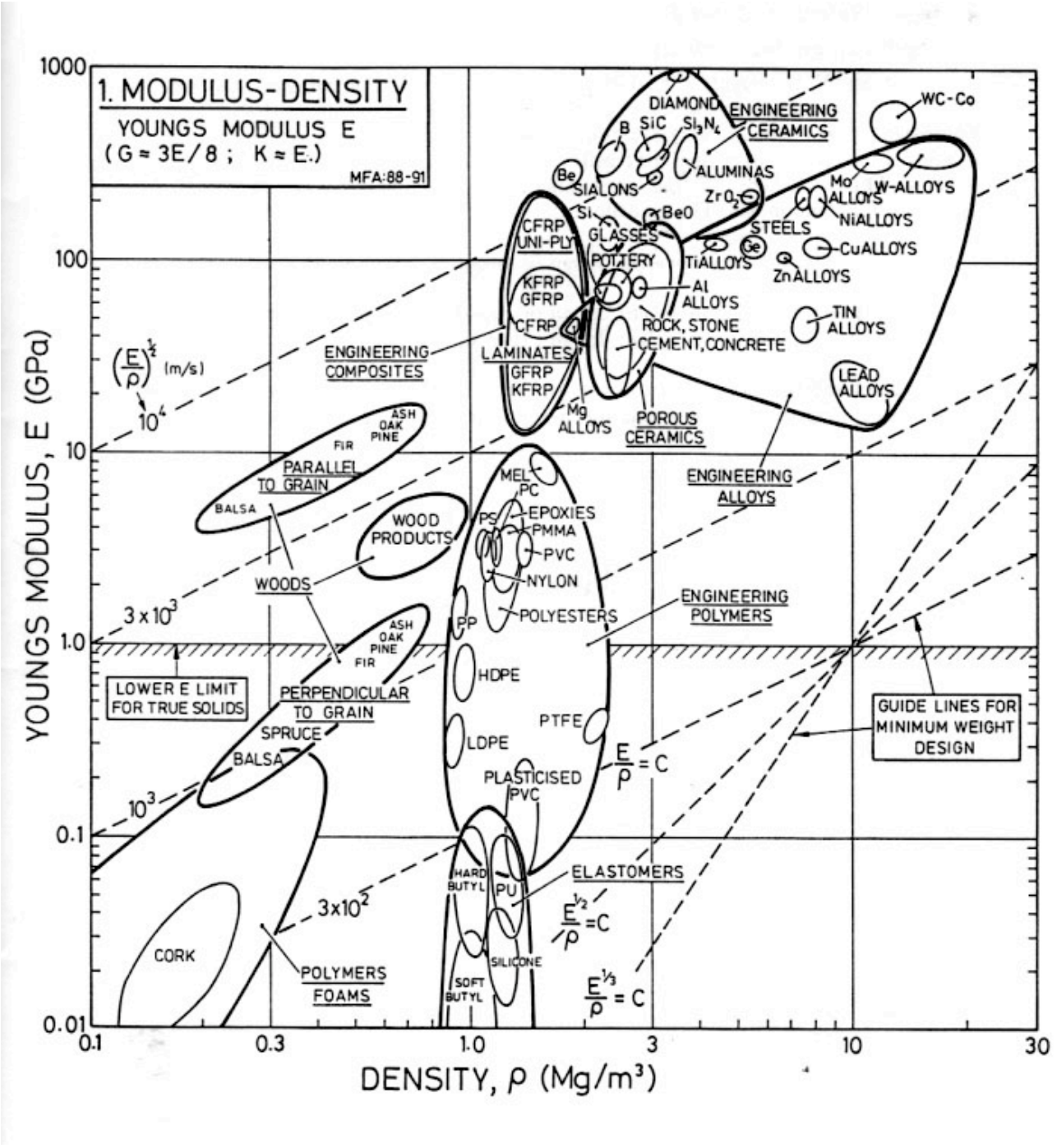
$$\delta = \frac{2ML^2}{\pi R^4 E}$$

where E is the Young's modulus.

b) Choose between the following material choices to select the most promising material to construct a wing for a small Unmanned Aerial Vehicle (which will carry load as a cantilever beam)

Material	**Density, ρ,** (Mg/m^3)	**Modulus, E,** (GPa)	**Poisson's Ratio**	**Yield Stress, σ_y,** (MPa)	**Price , p,** ($/Mg)
Mild Steel	7.9	203	0.31	220	375
Aluminum alloy (2024)	2.8	71	0.33	350	1650
Titanium alloy Ti-6Al4V	4.5	120	0.34	850	30000
Carbon fiber composite	1.5	230	0.3	1050	70000
Polyethelene (High Density)	0.96	1.1	0.39	30	1000
Wood (pine)	0.6	12	0.35	300	300
Silicon Carbide (SiC)	3.0	410	0.17	300 (Flexural)	30000

c) To a first approximation the human hip joint carries forces like a cantilever beam. In hip replacement surgery it is important to use material with the same total mass and bending stiffness as the bone that is removed. Bone has a modulus of 18 GPa and a density of 1.55 Mg/m^3. By reference to the attached materials selection chart suggest two materials that could be used for hip replacements. Include the chart, and your working with your answers.



From: Material Selection in Mechanical Design, M.F Ashby, Pergamon Press, Oxford, 1992