Problem M15 (Materials and Structures)

i) By considering the change in volume of an infinitessimal element undergoing small elongational strains show that the volumetric strain $\left(\frac{\Delta V}{V}\right) = \varepsilon_1 + \varepsilon_2 + \varepsilon_3$

ii) A continuous body experiences a displacement field, u_n that is described by:

$$u_1 = \left[0.5\left(x_1^2 - x_2^2\right) + 0.5 x_1 x_2\right] 10^{-3}\, mm$$

$$u_1 = \left[0.25\left(x_1^2 - x_2^2\right) - x_1 x_2\right] 10^{-3}\, mm$$

$$u_3 = 0.$$

Determine:

a) The 6 components of the strain tensor as a function of position (i.e. in terms of x_1, x_2, x_3)

b) The rigid body rotation about x_3 as a function of position (i.e. in terms of x_1, x_2, x_3).

c) The principal strains and the principal strain directions at x_1= 5mm and x_2=7 mm.

d) The volumetric strain at x_1= 5mm and x_2=7 mm.