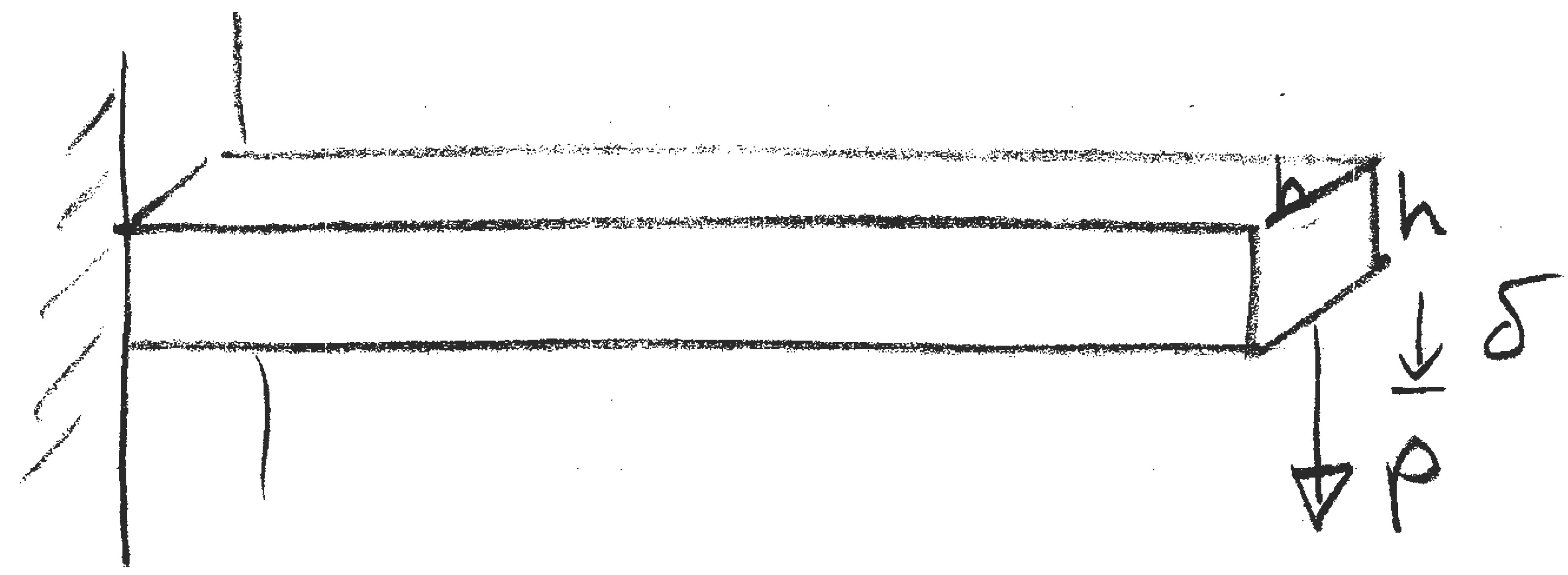
M6



For cantilever beam $\delta = \frac{PL^3}{3EI}$, $\sigma_{max} = \frac{PL}{I}\frac{h}{2}$

$I = \frac{1}{12} h^4$

Mass of beam $= \rho L A = \rho L h^2 = m$

(ρ = density)

~~Max~~ Stiffness of beam $= \frac{P}{\delta} = \frac{3EI}{L^3} = \frac{3}{12}\frac{h^4 E}{L^3}$

eliminate h^2 $\quad h^2 = \frac{m}{\rho L}$

$\therefore$ Stiffness $= \frac{P}{\delta} = \frac{1}{4}\left(\frac{m}{\rho L}\right)^2 E = K$

max stiffness for given mass (or vice-versa)

requires max $\frac{E}{\rho^2}$ $\Leftarrow$

For strength: $P_{max} = \dfrac{2\sigma_f I}{L h} = \dfrac{\sigma_f h^{\not{4}\,3}}{6 L \not{h}}$

$$P_{max} = \frac{\sigma_f h^3}{6L}$$

mass of beam, $m = \rho h^2 L$

eliminate h

$$P_{max} = \frac{\sigma_f}{6L}\left(\frac{m}{\rho L}\right)^{3/2}$$

so max P_{max} for given mass requires

max $\dfrac{\sigma_f}{\rho^{3/2}}$ ⇐

	E/GPa	σ_f/MPa	ρ/kg m^{-3}	E/ρ^2	$\sigma_f/\rho^{3/2}$
steel	193	1435	7900	3092	2044
Al	71	350	2800	9056	2362
Ti	120	850	4500	5926	2815
CFRP	70	700	1500	31111	12050 ⇐
Wood	12	50	600	33333	3402
Si	410	300	3000	45000 ⇐?	1826

c) a) SiC has the highest E/ρ^2, but may be impractical because of its low toughness

better choices might be wood or CFRP

b) CFRP has the highest $\sigma_f/\rho^{3/2}$

other factors – toughness, environmental durability, cost

d I & box sections are structurally efficient because they move material away from the neutral axis of the beam.

The key issue in selecting the shape of a beam is the ability to manufacture the shape in question.