Massachusetts Institute of Technology
Department of Aeronautics and Astronautics
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16.01/16.02 Unified Engineering I, II
Fall 2003

Problem Set 12

Name: ________________________

Due Date: 11/25/03

	Time Spent (min)
F12	
F13	
F14	
M13	
M14	
M15	
M16	
Study Time	

Announcements:

F12. For the two flows given by ...

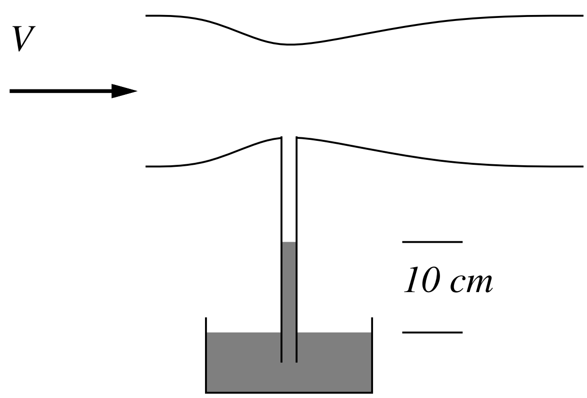
$$\psi(x,y) = \arctan(y/x) \qquad\qquad \phi(x,y) = x^2 + y^2$$

a) Determine the velocity fields, and sketch the streamlines.

b) Determine the volume flow rate through a circle of radius r.

c) Which of these flows is not feasible to set up in a lab? Explain.

F13. A venturi has a minimum throat area of 0.7 times the inlet/outlet areas. The water tank is open to ambient atmospheric pressure. Determine the sea-level wind speed V which is needed at the inlet to raise the water column 10 cm.



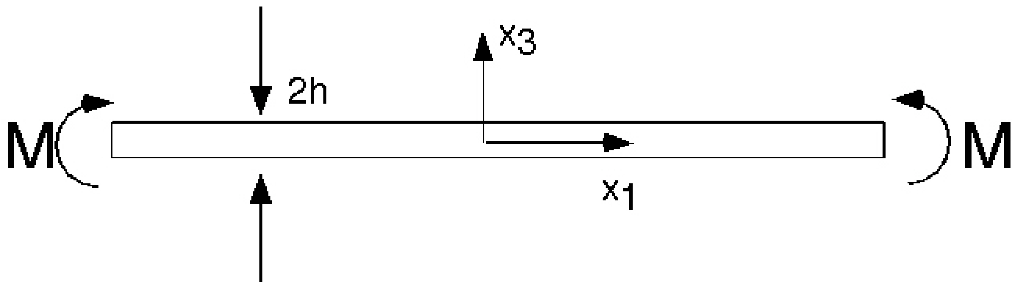
F14. $\phi_1(x,y)$ and $\phi_2(x,y)$ are known to be physically-possible flows (i.e. satisfy mass conservation), and their corresponding pressure fields $p_1(x,y)$ and $p_2(x,y)$ are known via the Bernoulli equation.

a) A third flow is now defined by $\phi_3(x,y) = \phi_1 + \phi_2$. Explain how you would obtain its corresponding pressure field p_3.

b) Yet another flow $\phi_4 = \partial\phi_1/\partial x$ is defined. Is this a physically-possible flow?

Problem M13 (Materials and Structures)

a) The state of axial stress through the thickness of a beam in pure bending (i.e. loaded only by a moment) is given by:



$$\sigma_{11} = \left(\frac{M}{I}\right) x_3 \qquad \text{for } -h < x_3 < +h$$

M is the bending moment (which is constant in x_1), and I is the second moment of area of the cross-section of the beam (which is also constant). If $\sigma_{12} = \sigma_{32} = \sigma_{22} = \sigma_{33} = 0$ what can you say about the variation of the shear stress σ_{13} with x_1, x_2 and x_3?

b) The bending moment M now varies as a function of x_1 according to $M = cx_1$ The axial stress is still give by $\sigma_{11} = \left(\frac{M}{I}\right) x_3$. Again, $\sigma_{12} = \sigma_{32} = \sigma_{22} = \sigma_{33} = 0$. How does σ_{13} vary with x_1, x_2 and x_3? Note $\sigma_{13} = 0$ for $-h = x_3$ and $x_3 = +h$ (the top and bottom surfaces of the bean are free surfaces and do not have any stress acting on them).

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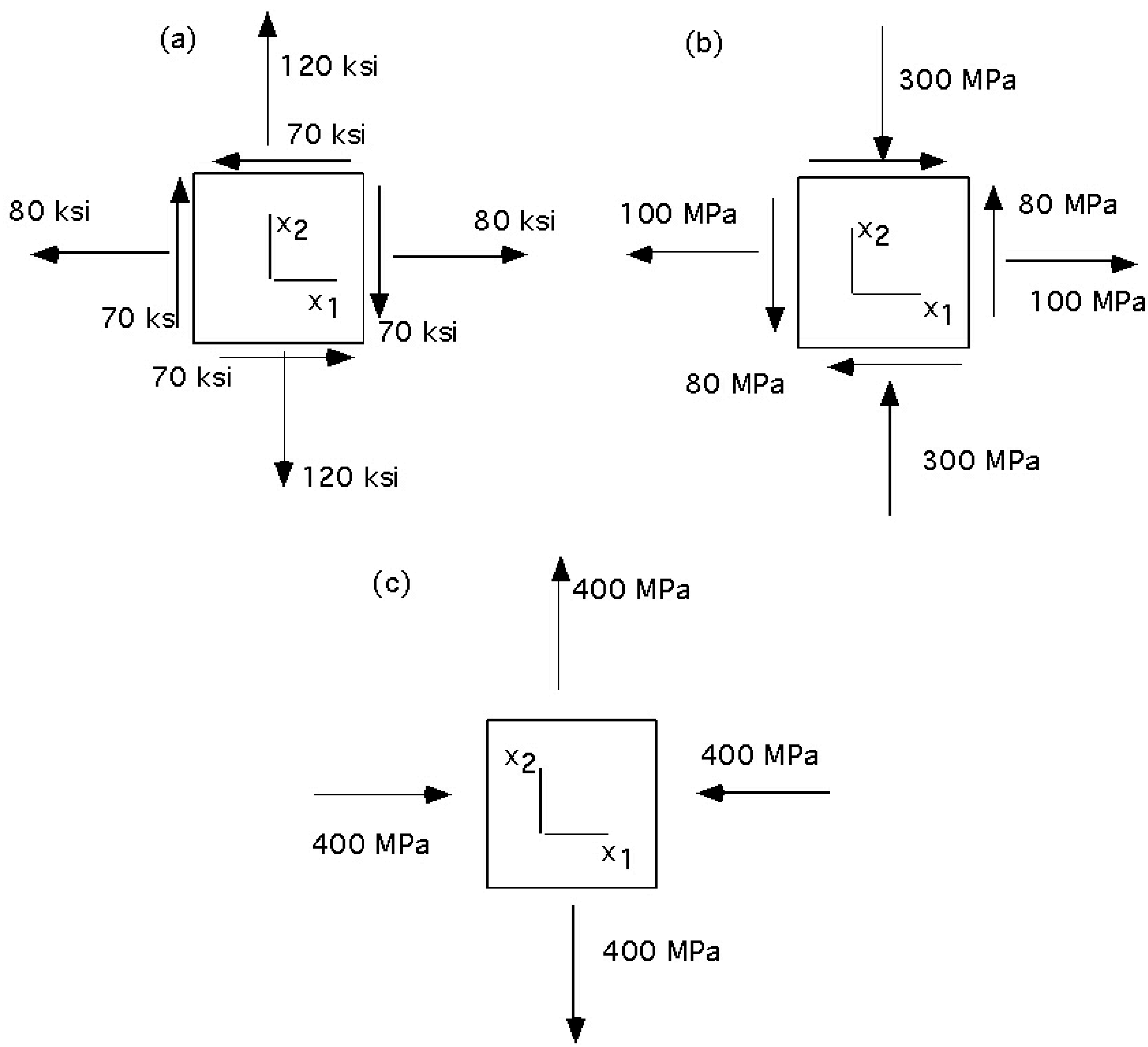
Problem M14

For each of the states of plane stress shown acting on the differential elements drawn below do the following:

1) Draw a Mohr's circle describing the stress state.

2) Determine the principal stresses and the maximum shear stress

3) Calculate the angle from the x_1 direction as shown to the more tensile principal stress direction. Note whether the angle is clockwise or counterclockwise.



Problem M15 (Materials and Structures)

i) By considering the change in volume of an infinitessimal element undergoing small elongational strains show that the volumetric strain $\left(\frac{\Delta V}{V}\right) = \varepsilon_1 + \varepsilon_2 + \varepsilon_3$

ii) A continuous body experiences a displacement field, u_n that is described by:

$$u_1 = \left[0.5\left(x_1^2 - x_2^2\right) + 0.5 x_1 x_2\right] 10^{-3} \, mm$$

$$u_1 = \left[0.25\left(x_1^2 - x_2^2\right) - x_1 x_2\right] 10^{-3} \, mm$$

$$u_3 = 0.$$

Determine:

a) The 6 components of the strain tensor as a function of position (i.e. in terms of x_1, x_2, x_3)

b) The rigid body rotation about x_3 as a function of position (i.e. in terms of x_1, x_2, x_3).

c) The principal strains and the principal strain directions at x_1= 5mm and x_2=7 mm.

d) The volumetric strain at x_1= 5mm and x_2=7 mm.

Problem M16

The purpose of this question is to demonstrate the equivalence of the two methods at our disposal for transforming strain (and stress) and for calculating the principal values and directions.

Given a state of plane strain: ε_{11} = -0.000200, ε_{22} = +0.000400, ε_{12} = -0.000200, do the following:

a) Draw a Mohr's circle for the strain state. Note you may find it convenient to work in terms of "micro-strain" (strain/10^{-6})

b) From the Mohr's circle determine the principal strains ε_I and ε_{II} and principal directions, $\tilde{x}_1$ and $\tilde{x}_2$. You should specify the directions as counterclockwise angles with respect to the original x_1 and x_2 coordinate system.

c) Determine the direction cosines for the transformation between x_1,x_2 and $\tilde{x}_1$,$\tilde{x}_2$

d) Using the appropriate tensor operation show that the original strain tensor (ε_{nm}) transforms to the principal strain tensor ($\tilde{\varepsilon}_{pq}$).