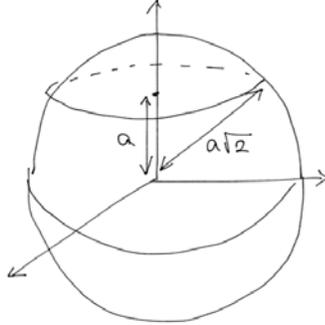


Problems: Spherical Coordinates

1. Find the volume of a solid spherical cap obtained by slicing a solid sphere of radius $a\sqrt{2}$ by a plane at a distance a from the center. (See picture.)



Answer: In session 76 we found the limits:

inner ρ : $a/\cos\phi$ to $a\sqrt{2}$,

middle ϕ : 0 to $\pi/4$,

outer θ : 0 to 2π .

$$\text{Volume} = \int_0^{2\pi} \int_0^{\pi/4} \int_{a/\cos\phi}^{a\sqrt{2}} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta.$$

$$\text{Inner: } \frac{1}{3}\rho^3 \sin\phi \Big|_{a/\cos\phi}^{a\sqrt{2}} = \frac{2a^3\sqrt{2}}{3} \sin\phi - \frac{a^3 \sin\phi}{3 \cos^3\phi}.$$

$$\text{Middle: } \left[-\frac{2a^3\sqrt{2}}{3} \cos\phi - \frac{a^3}{6 \cos^2\phi} \right]_{\phi=0}^{\pi/4} = -\frac{2a^3}{3} - \frac{a^3}{3} - \left(-\frac{2\sqrt{2}a^3}{3} - \frac{a^3}{6} \right) = \frac{2\sqrt{2}a^3}{3} - \frac{5a^3}{6}.$$

$$\text{Outer: } 2\pi \left(\frac{2\sqrt{2}a^3}{3} - \frac{5a^3}{6} \right) = \frac{a^3\pi}{3} (4\sqrt{2} - 5) \approx 0.7a^3.$$

The volume of the entire sphere is about $12a^3$ and we're looking at approximately the top sixth of its height. The sphere has more volume near its midpoint than at its top and bottom, so this answer seems reasonable.

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