

Problems: Jacobian for Spherical Coordinates

Use the Jacobian to show that the volume element in spherical coordinates is the one we've been using.

Answer: $z = \rho \cos \phi$, $x = \rho \sin \phi \cos \theta$, $y = \rho \sin \phi \sin \theta$

$$\begin{aligned} \Rightarrow \frac{\partial(x, y, z)}{\partial(\rho, \phi, \theta)} &= \begin{vmatrix} \sin \phi \cos \theta & \rho \cos \phi \cos \theta & -\rho \sin \phi \sin \theta \\ \sin \phi \sin \theta & \rho \cos \phi \sin \theta & \rho \sin \phi \cos \theta \\ \cos \phi & -\rho \sin \phi & 0 \end{vmatrix} \\ &= \cos \phi \begin{vmatrix} \rho \cos \phi \cos \theta & -\rho \sin \phi \sin \theta \\ \rho \cos \phi \sin \theta & \rho \sin \phi \cos \theta \end{vmatrix} + \rho \sin \phi \begin{vmatrix} \sin \phi \cos \theta & -\rho \sin \phi \sin \theta \\ \sin \phi \sin \theta & \rho \sin \phi \cos \theta \end{vmatrix} \\ &= \cos \phi (\rho^2 \sin \phi \cos \phi) + \rho \sin \phi \rho \sin^2 \phi \\ &= \rho^2 \sin \phi (\cos^2 \phi + \sin^2 \phi) \\ &= \rho^2 \sin \phi \quad (\text{as promised.}) \end{aligned}$$

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