

## Chain rule with more variables

1. Let  $w = xyz$ ,  $x = u^2v$ ,  $y = uv^2$ ,  $z = u^2 + v^2$ .

a) Use the chain rule to find  $\frac{\partial w}{\partial u}$ .

b) Find the total differential  $dw$  in terms of  $du$  and  $dv$ .

c) Find  $\frac{\partial w}{\partial u}$  at the point  $(u, v) = (1, 2)$ .

**Answer:** a) The chain rule says

$$\begin{aligned}\frac{\partial w}{\partial u} &= \frac{\partial w}{\partial x} \frac{\partial x}{\partial u} + \frac{\partial w}{\partial y} \frac{\partial y}{\partial u} + \frac{\partial w}{\partial z} \frac{\partial z}{\partial u} \\ &= (yz)(2uv) + (xz)(v^2) + (xy)(2u).\end{aligned}$$

b) Using the formulas given we get

$$dw = yz dx + xz dy + xy dz$$

and

$$dx = 2uv du + u^2 dv, \quad dy = v^2 du + 2uv dv, \quad dz = 2u du + 2v dv.$$

Substituting for  $dx$ ,  $dy$ ,  $dz$  in the equation for  $dw$  gives

$$\begin{aligned}dw &= (yz)(2uv du + u^2 dv) + (xz)(v^2 du + 2uv dv) + (xy)(2u du + 2v dv). \\ &= (2yzuv + xzv^2 + 2xyu) du + (yzu^2 + 2xzuv + 2xyv) dv.\end{aligned}$$

Therefore

$$\frac{\partial w}{\partial u} = 2yzuv + xzv^2 + 2xyu \quad \text{and} \quad \frac{\partial w}{\partial v} = yzu^2 + 2xzuv + 2xyv.$$

c) We do the chain of computations to compute the partial.

$$(u, v) = (1, 2) \Rightarrow (x, y, z) = (2, 4, 5) \Rightarrow \frac{\partial w}{\partial u} = (20)(4) + (10)(4) + (8)(2) = 136.$$

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