

Chain Rule

1. The temperature on a hot surface is given by

$$T = 100 e^{-(x^2+y^2)}.$$

A bug follows the trajectory $\mathbf{r}(t) = \langle t \cos(2t), t \sin(2t) \rangle$.

a) What is the rate that temperature is changing as the bug moves?

b) Draw the level curves of T and sketch the bug's trajectory.

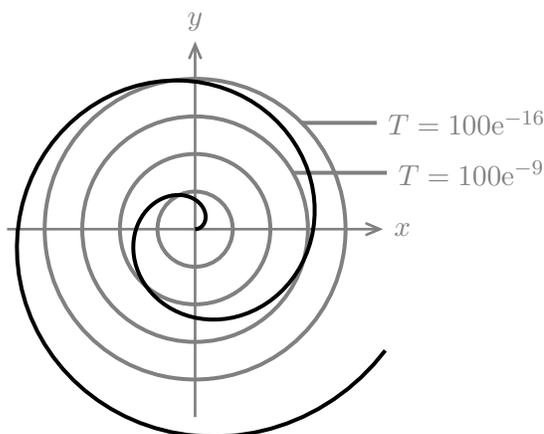
Answer: a) The chain rule says

$$\begin{aligned} \frac{dT}{dt} &= \frac{\partial T}{\partial x} \frac{dx}{dt} + \frac{\partial T}{\partial y} \frac{dy}{dt} \\ &= -200xe^{-(x^2+y^2)}(\cos(2t) - 2t \sin(2t)) - 200ye^{-(x^2+y^2)}(\sin(2t) + 2t \cos(2t)). \end{aligned}$$

You could stop here, or substitute $x = t \cos(2t)$ and $y = t \sin(2t)$. After simplification you get

$$\frac{dT}{dt} = -200 t e^{-t^2}.$$

b) The level curves of T are the curves $x^2 + y^2 = \text{constant}$, i.e., circles. The bug moves in a spiral.



2. Suppose $w = f(x, y)$ and $x = t^2$, $y = t^3$. Suppose also that at $(x, y) = (1, 1)$ we have $\frac{\partial w}{\partial x} = 3$ and $\frac{\partial w}{\partial y} = 1$. Compute $\frac{dw}{dt}$ at $t = 1$.

Answer: At $t = 1$ we have $(x, y) = (1, 1)$, $\left. \frac{dx}{dt} \right|_1 = 2$, $\left. \frac{dy}{dt} \right|_1 = 3$. Therefore the chain rule says

$$\left. \frac{dw}{dt} \right|_1 = \left. \frac{\partial f}{\partial x} \right|_{(1,1)} \left. \frac{dx}{dt} \right|_1 + \left. \frac{\partial f}{\partial y} \right|_{(1,1)} \left. \frac{dy}{dt} \right|_1 = 3(2) + 1(3) = 9.$$

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18.02SC Multivariable Calculus
Fall 2010

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