

Problems: Limits in Spherical Coordinates

1. Find the limits needed to use spherical coordinates to compute the volume of a sphere of radius a .

Answer: Limits: inner ρ : 0 to a –radial segments
 middle ϕ : 0 to π –fan of rays.
 outer θ : 0 to 2π –volume.

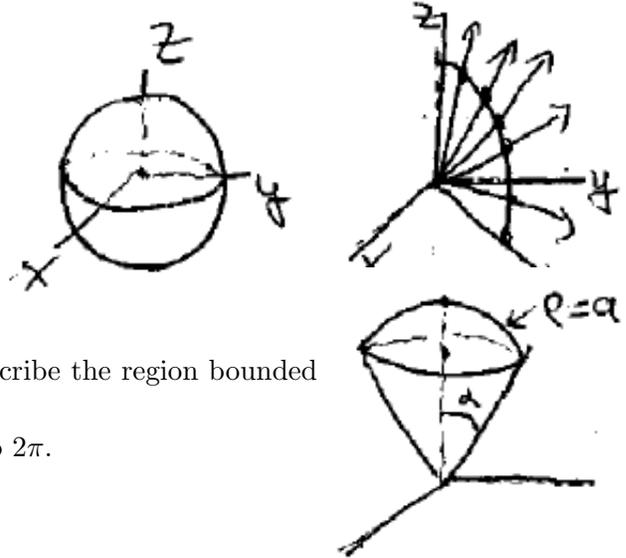
To set up and evaluate the integral (optional):

$$V = \iiint_D dV = \int_0^{2\pi} \int_0^\pi \int_0^a \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta$$

$$\text{Inner: } \left. \frac{\rho^3}{3} \sin \phi \right|_0^a = \frac{a^3}{3} \sin \phi$$

$$\text{Middle: } \left. -\frac{a^3}{3} \cos \phi \right|_0^\pi = \frac{2}{3} a^3$$

Outer: $\frac{4}{3}\pi a^3$ –as it should be.



2. Find limits in spherical coordinates which describe the region bounded by the sphere $\rho = a$ and the cone $\phi = \alpha$.

Answer: Limits: ρ : 0 to a , ϕ : 0 to α , θ : 0 to 2π .

3. Find limits for a solid spherical cap obtained by slicing a solid sphere of radius $a\sqrt{2}$ by a plane at a distance a from the center.

Answer: See the picture.

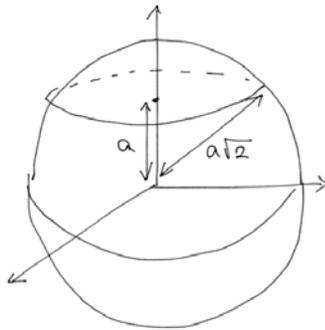


Figure 1: Sphere of radius $a\sqrt{2}$ sliced by the plane $z = a$.

Inner ρ : $a/\cos \phi$ to $a\sqrt{2}$, middle ϕ : 0 to $\pi/4$, outer θ : 0 to 2π .

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