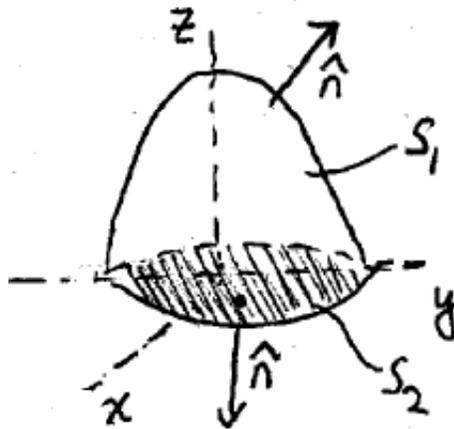


## Problems: Divergence Theorem

Let  $S_1$  be the part of the paraboloid  $z = 1 - x^2 - y^2$  which is above the  $xy$ -plane and  $S_2$  be the unit disk in the  $xy$ -plane. Use the divergence theorem to find the flux of  $\mathbf{F}$  upward through  $S_1$ , where  $\mathbf{F} = \langle yz, xz, xy \rangle$ .

**Answer:** Write  $\mathbf{F} = M\mathbf{i} + N\mathbf{j} + P\mathbf{k}$ , where  $M = yz$ ,  $N = xz$ , and  $P = xy$ . Then

$$\operatorname{div}\mathbf{F} = M_x + N_y + P_z = 0.$$



The divergence theorem says:  $\text{flux} = \iint_{S_1+S_2} \mathbf{F} \cdot \mathbf{n} dS = \iiint_D \operatorname{div}\mathbf{F} dV = \iiint_D 0 dV = 0$

$$\Rightarrow \iint_{S_1} \mathbf{F} \cdot \mathbf{n} dS + \iint_{S_2} \mathbf{F} \cdot \mathbf{n} dS = 0 \Rightarrow \iint_{S_1} \mathbf{F} \cdot \mathbf{n} dS = - \iint_{S_2} \mathbf{F} \cdot \mathbf{n} dS.$$

Therefore to find what we want we only need to compute the flux through  $S_2$ .

But  $S_2$  is in the  $xy$ -plane, so  $dS = dx dy$ ,  $\mathbf{n} = -\mathbf{k} \Rightarrow \mathbf{F} \cdot \mathbf{n} dS = -xy dx dy$  on  $S_2$ .

Since  $S_2$  is the unit disk, symmetry gives

$$\iint_{S_2} -xy dx dy = 0 \Rightarrow \iint_{S_1} \mathbf{F} \cdot \mathbf{n} dS = - \iint_{S_2} \mathbf{F} \cdot \mathbf{n} dS = 0.$$

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