Intersection of a line and a plane

1. Consider the plane $\mathcal{P} = 2x + y - 4z = 4$.

a) Find all points of intersection of $\mathcal{P}$ with the line

$$x = t, \quad y = 2 + 3t, \quad z = t.$$

b) Find all points of intersection of $\mathcal{P}$ with the line

$$x = 1 + t, \quad y = 4 + 2t, \quad z = t.$$

c) Find all points of intersection of $\mathcal{P}$ with the line

$$x = t, \quad y = 4 + 2t, \quad z = t.$$

Answer: a) To find the intersection we substitute the formulas for x, y and z into the equation for $\mathcal{P}$ and solve for t.

$$2(t) + (2 + 3t) - 4(t) = 4 \iff t = 2.$$

Now use $t = 2$ to find the point of intersection: $(x, y, z) = (2, 8, 2)$.

b) Substituting gives

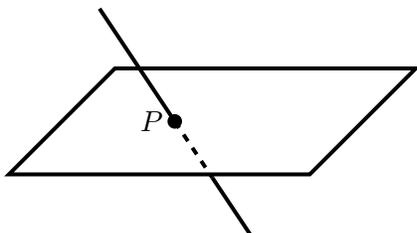$$2(1 + t) + (4 + 2t) - 4(t) = 4 \iff 6 = 4. \iff \text{no values of } t \text{ satisfy this equation.}$$

There are no points of intersection.

c) Substituting gives

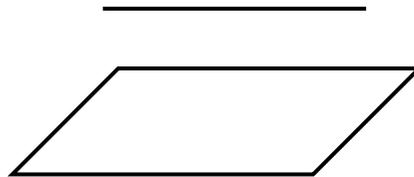$$2(t) + (4 + 2t) - 4(t) = 4 \iff 4 = 4. \iff \text{all values of } t \text{ satisfy this equation.}$$

The line is contained in the plane, i.e., all points of the line are in its intersection with the plane.

Here are cartoon sketches of each part of this problem.

(a) line intersects the plane in a point

(b) line is parallel to the plane

(c) line is in the plane

18.02SC Multivariable Calculus
Fall 2010