

## Problems: Practice with Triple Integrals

Find the moment of inertia about the  $z$ -axis of a solid bounded by the paraboloid  $z = x^2 + y^2$  and the plane  $z = 1$ . Assume the solid has uniform density 1.

**Answer:** We use the formula  $I = \iiint \rho r^2 dV$  with density  $\rho = 1$ . Converting to polar coordinates, the equation of the paraboloid becomes  $z = r^2$  and we get the limits of integration  $0 \leq r \leq \sqrt{z}$ .

$$\begin{aligned} I &= \iiint_{\text{solid}} \rho r^2 dV \\ &= \int_0^1 \int_0^{2\pi} \int_0^{\sqrt{z}} r^2 \cdot r dr d\theta dz \\ &= \int_0^1 \int_0^{2\pi} \frac{z^2}{4} d\theta dz \\ &= \int_0^1 \pi \frac{z^2}{2} dz \\ &= \frac{\pi}{6}. \end{aligned}$$

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