

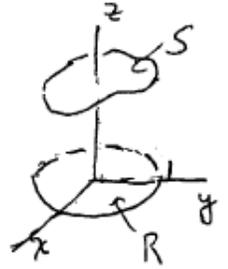
## Problems: Stokes' Theorem

1. Let  $\mathbf{F} = x^2\mathbf{i} + x\mathbf{j} + z^2\mathbf{k}$  and let  $S$  be the graph of  $z = x^3 + xy^2 + y^4$  over the unit disk. Use Stokes' Theorem to compute  $\oint_C \mathbf{F} \cdot d\mathbf{r}$ , where  $C$  is the boundary of  $S$ .

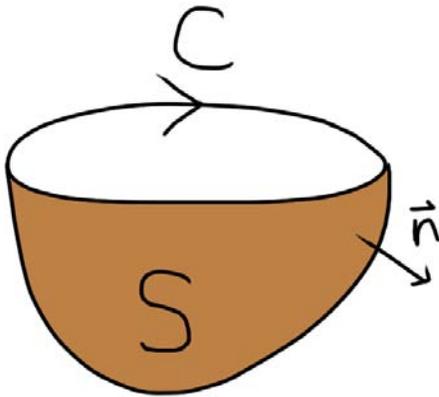
**Answer:**  $\text{curl}\mathbf{F} = \langle 0, 0, 1 \rangle$ ,  $\mathbf{n} dS = \langle -z_x, -z_y, 1 \rangle dx dy \Rightarrow \text{curl}\mathbf{F} \cdot \mathbf{n} dS = dx dy$ .

$$\Rightarrow \iint_S \text{curl}\mathbf{F} \cdot \mathbf{n} dS = \iint_R dx dy = \text{area } R = \pi.$$

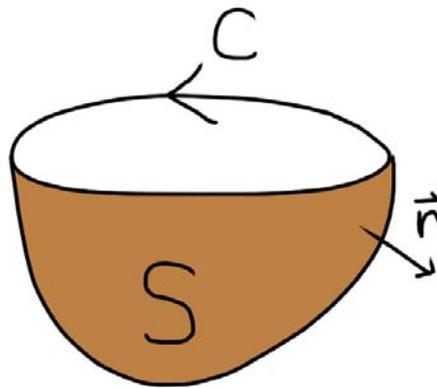
Therefore, by Stokes' Theorem  $\oint_C \mathbf{F} \cdot d\mathbf{r} = \pi$ .



2. Which of the figures below shows a compatibly oriented surface and curve?



(a)



(b)

**Answer:** On surface (a), the curve  $C$  is oriented compatibly with the surface  $S$  shown. To make this easier to see, add more arrows indicating the orientation of  $C$ .

MIT OpenCourseWare  
<http://ocw.mit.edu>

18.02SC Multivariable Calculus  
Fall 2010

For information about citing these materials or our Terms of Use, visit: <http://ocw.mit.edu/terms>.