

General Formula for $\mathbf{n} dS$

Suppose S is a surface parametrized by x and y and \mathbf{N} is any vector normal to S (not necessarily unit length). Then $\mathbf{n} dS = \frac{\mathbf{N}}{\mathbf{N} \cdot \mathbf{k}} dx dy$. Here \mathbf{n} is the upward unit normal

Example: for the sphere $x^2 + y^2 + z^2 = a^2$ with $\mathbf{N} = \langle x, y, z \rangle$, find $\mathbf{n} dS$.

Answer: $\mathbf{n} dS = \frac{\mathbf{N}}{\mathbf{N} \cdot \mathbf{k}} dx dy = \left\langle \frac{x}{z}, \frac{y}{z}, 1 \right\rangle dx dy$.

(Just like if we wrote $z = \sqrt{a^2 - x^2 - y^2}$, $\mathbf{n} dS = \langle -z_x, -z_y, 1 \rangle dx dy$.)



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18.02SC Multivariable Calculus
Fall 2010

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