

**CHRISTINE
BREINER:**

Welcome back to recitation. In this video, I'd like us to work on the following problem that has to do with tangent planes and approximations.

So we know that a rectangle-- we'll say a rectangle has sides x and y , and we know that to find the area of the rectangle then, we just take x times y , and I would like us to approximate the area for x equal to 2.1 and y equal to 2.8.

And obviously, with this type of equation, it's not hard to just compute this, but I'd like us to use the tangent plane approximation to determine the value, and then we'll compare it to the actual value, just to give us an idea of how we can use the tangent plane to approximate things. And obviously, I'd like to do this near x equal 2 and y equal 3. So when you're doing your tangent plane approximation, do the approximation at x equal 2 and y equal 3.

And then when you're done with that, I would like you to answer this question. So near x equal 2 and y equal 3, which has the greater effect? A change in x or an equal change in y ? So if I change by x in the same value as I change by y , which will have the greater impact? So why don't you pause the tape, work on the problems, and when you're ready to see how I do them, bring it back up, and I'll come back and show you.

OK, welcome back. So we're going to work on this problem, and as I mentioned, the first thing we want to do is do a tangent plane approximation near x equal 2 and y equal 3. So let me write down what we're going to need in order to do that.

First, we'll remind ourselves that in this case, what I'm going to approximate is the area function for a rectangle, which is A of x, y is equal to x times y . So that's the function I'm going to be approximating. And now the actual approximation, the tangent plane approximation, has the following form.

So we know A of x, y is approximately-- well, it's going to be the area evaluated at the point I'm interested in, which we said was 2 comma 3, right, plus the x -derivative of area evaluated at 2 comma 3, times the change in x . And the change in x is where x is now versus where we started, which is at x equal 2. And then the same thing in terms of y . So you do the y -derivative evaluated at 2 comma 3, and then you do the change in y . And since we started at y equals 3, the change in y is y minus 3, OK?

So now we obviously need to fill in three quantities here. The three things we need to

evaluate: area evaluated at $(2, 3)$, its x -derivative evaluated at $(2, 3)$, and its y -derivative evaluated at $(2, 3)$. So let me just point out what we have.

Area evaluated at $(2, 3)$, well, that's just 2 times 3, which we can do, so that's 6. That's pretty easy. Now A_x , so the derivative of the area function with respect to x . The derivative of the area function with respect to x is just y . y in that case we treat as a constant. We take this derivative. We just get y back. And so, A_x evaluated at $(2, 3)$ is going to be y , which-- y here is 3, so we get A_x is equal to 3 at $(2, 3)$.

In a similar vein, we can immediately look and see that A_y evaluated at $(2, 3)$ is going to be 2. And the reason for that, of course, is if we look back here, the derivative of A with respect to y is x , so evaluating it at $(2, 3)$ gives us 2.

OK. So now we just have to fill everything in. So my tangent plane approximation now says I get 6 plus 3 times x minus 2 plus 2 times y minus 3. And what I wanted was the area at 2.1 comma 2.8. So if I fill in those values for x and y -- so 2.1 is x and 2.8 is y -- if I fill those in, I get that this is equal to-- well, I'll keep writing approximately just to be safe-- is 6 plus 3 times-- well, 2.1 minus 2 gives me a 0.1 there and 2 times 2.8 minus 3 gives me a negative 0.2 there, so I get a negative 0.4, I get a positive 0.3, so together this is a negative 0.1. 6 minus 0.1 gives me 5.9. So the area based on the approximation is 5.9 square units. And, in actuality, if you multiply it out, I think you get something like 5.88, so the approximation is very good, right? We're close to $(2, 3)$ and that is one of the reasons we can know it's going to be very, it should be very good. It should be pretty good, OK?

Now, I just have to answer the second part of the question. So let me remind us what the second part was. It was near x equal 2 and y equal 3, which has a greater effect? A change in x or an equal change in y ? And to do that, it's really easiest to come back and look at maybe this line. I'll underline this line right here. Actually, I'll box it, OK?

This value will represent the change in x . So we started at 2, and we go somewhere, and that'll represent the change in x . This value represents the change in y . So if we look at which has a greater impact near $(2, 3)$, if my change in x and my change in y are equal, then obviously this term has a bigger impact, because there's a 3. The coefficient here is 3 and the coefficient here is 2. And so the point is, changes in x will have a greater effect than changes in y .

And what this corresponds to pictorially is if we make a slice where we keep y constant and we

look at the curve in the xz -plane, that corresponds to the fact that the derivative in the xz -plane of the curve there is more significant-- the curve there has a steeper derivative-- than if I kept the x -value fixed and I looked in the yz -plane and I looked at the curve there. So the sort of one-dimensional picture is that the derivative of a curve in the x -direction, keeping y fixed, is steeper than the derivative of the curve in the y -direction, keeping x fixed.

So maybe-- hopefully, that wasn't more confusing than it should have been. From here, you can see it right away, but that's sort of the picture of it as well. So I think that's where I'll stop.