

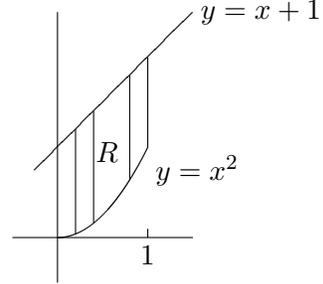
Problems: Regions of Integration

1. Find the mass of the region R bounded by $y = x + 1$; $y = x^2$; $x = 0$ and $x = 1$, if density $= \delta(x, y) = xy$.

Answer:

Inner limits: y from x^2 to $x + 1$. Outer limits: x from 0 to 1.

$$\Rightarrow M = \int \int_R \delta(x, y) dA = \int_{x=0}^1 \int_{y=x^2}^{x+1} xy dy dx$$

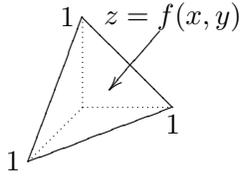


$$\text{Inner: } \int_{x^2}^{x+1} xy dy = x \frac{y^2}{2} \Big|_{x^2}^{x+1} = \frac{x(x+1)^2}{2} - \frac{x^5}{2} = \frac{x^3}{2} + x^2 + \frac{x}{2} - \frac{x^5}{2}.$$

$$\text{Outer: } \int_0^1 \left(\frac{x^3}{2} + x^2 + \frac{x}{2} - \frac{x^5}{2} \right) dx = \frac{x^4}{8} + \frac{x^3}{3} + \frac{x^2}{4} - \frac{x^6}{12} \Big|_0^1 = \frac{1}{8} + \frac{1}{3} + \frac{1}{4} - \frac{1}{12} = \frac{5}{8}.$$

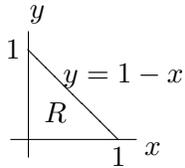
Note: The syntax $y = x^2$ in limits is redundant but useful. We know it must be y because of the dy matching the integral sign.

2. Find the volume of the tetrahedron shown below.



Tetrahedron

Answer: The surface has height: $z = 1 - x - y$.



Region R

$$\text{Limits: inner: } 0 < y < 1 - x, \quad \text{outer: } 0 < x < 1. \quad \Rightarrow V = \int_{x=0}^1 \int_{y=0}^{1-x} (1 - x - y) dy dx.$$

$$\text{Inner: } \int_{y=0}^{1-x} (1 - x - y) dy = y - xy - \frac{y^2}{2} \Big|_0^{1-x} = 1 - x - x + x^2 - \frac{1}{2} + x - \frac{x^2}{2}.$$

$$\text{Outer: } \int_0^1 \left(\frac{1}{2} - x + \frac{x^2}{2} \right) dx = \frac{1}{2} - \frac{1}{2} + \frac{1}{6} = \frac{1}{6}.$$

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