

## Proofs using vectors

1. The median of a triangle is a vector from a vertex to the midpoint of the opposite side. Show the sum of the medians of a triangle =  $\mathbf{0}$ .

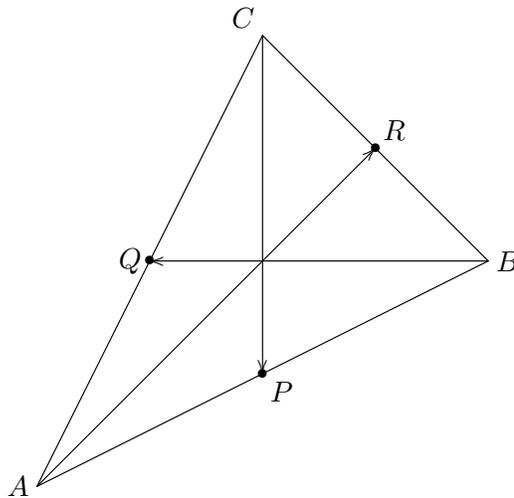
**Answer:** The median of side  $AB$  is the vector from vertex  $C$  to the midpoint of  $AB$ . Label this midpoint as  $P$ . As usual we write  $\mathbf{P}$  for the origin vector  $\overrightarrow{\mathbf{OP}}$ .

The midpoint  $\mathbf{P} = \frac{1}{2}(\mathbf{A} + \mathbf{B}) \Rightarrow \overrightarrow{\mathbf{CP}} = \frac{1}{2}(\mathbf{B} + \mathbf{A}) - \mathbf{C}$ .

Likewise:  $\overrightarrow{\mathbf{BQ}} = \frac{1}{2}(\mathbf{A} + \mathbf{C}) - \mathbf{B}$  and  $\overrightarrow{\mathbf{AR}} = \frac{1}{2}(\mathbf{B} + \mathbf{C}) - \mathbf{A}$ .

$\Rightarrow$  sum of medians is

$$\overrightarrow{\mathbf{CP}} + \overrightarrow{\mathbf{BQ}} + \overrightarrow{\mathbf{AR}} = \left( \frac{1}{2}(\mathbf{B} + \mathbf{A}) - \mathbf{C} \right) + \left( \frac{1}{2}(\mathbf{A} + \mathbf{C}) - \mathbf{B} \right) + \left( \frac{1}{2}(\mathbf{B} + \mathbf{C}) - \mathbf{A} \right) = \mathbf{0}.$$



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