

## Cross product

1. a) Compute  $\langle 1, 3, 1 \rangle \times \langle 2, -1, 5 \rangle$ .

b) Compute  $(\mathbf{i} + 2\mathbf{j}) \times (2\mathbf{i} - 3\mathbf{j})$ .

**Answer:** a) We use the determinant method:

$$\langle 1, 3, 1 \rangle \times \langle 2, -1, 5 \rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 3 & 1 \\ 2 & -1 & 5 \end{vmatrix} = \mathbf{i}(16) - \mathbf{j}(3) + \mathbf{k}(-7) = \langle 16, -3, -7 \rangle$$

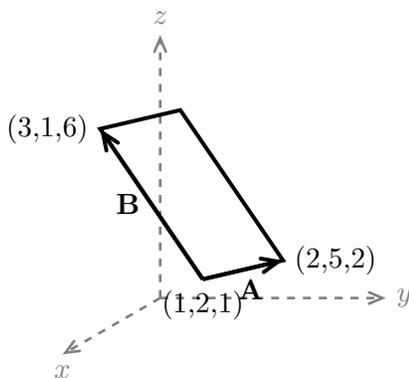
b) Using determinants we get

$$(\mathbf{i} + 2\mathbf{j}) \times (2\mathbf{i} - 3\mathbf{j}) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & 0 \\ 2 & -3 & 0 \end{vmatrix} = \mathbf{i}(0) - \mathbf{j}(0) + \mathbf{k}(-7) = -7\mathbf{k}.$$

Multiplying directly and using  $\mathbf{i} \times \mathbf{j} = \mathbf{k}$  etc we get

$$(\mathbf{i} + 2\mathbf{j}) \times (2\mathbf{i} - 3\mathbf{j}) = \mathbf{i} \times \mathbf{i} - 3\mathbf{i} \times \mathbf{j} + 4\mathbf{j} \times \mathbf{i} - 6\mathbf{j} \times \mathbf{j} = \mathbf{0} - 3\mathbf{k} - 4\mathbf{k} - 6 \cdot \mathbf{0} = -7\mathbf{k}.$$

2. Find the area of the parallelogram shown.



**Answer:** The area is  $|\mathbf{A} \times \mathbf{B}|$ , where  $\mathbf{A}$  and  $\mathbf{B}$  are the vectors along two adjacent edges of the parallelogram. These vectors are

$$\mathbf{A} = \langle 2, 5, 2 \rangle - \langle 1, 2, 1 \rangle = \langle 1, 3, 1 \rangle \quad \text{and} \quad \mathbf{B} = \langle 3, 1, 6 \rangle - \langle 1, 2, 1 \rangle = \langle 2, -1, 5 \rangle.$$

We computed this cross product in problem (1a). So,

$$\text{area} = |\langle 16, -3, -7 \rangle| = \sqrt{256 + 9 + 49} = \sqrt{314}.$$

MIT OpenCourseWare  
<http://ocw.mit.edu>

18.02SC Multivariable Calculus  
Fall 2010

For information about citing these materials or our Terms of Use, visit: <http://ocw.mit.edu/terms>.