

Tangent approximation

1. Find the equation of the tangent plane to the graph of $z = xy^2$ at the point $(1,1,1)$.

Answer: $\frac{\partial z}{\partial x} = y^2$ and $\frac{\partial z}{\partial y} = 2xy \Rightarrow \frac{\partial z}{\partial x}(1,1) = 1$ and $\frac{\partial z}{\partial y}(1,1) = 1$.

The tangent plane at $(1,1,1)$ is

$$(z - 1) = \left. \frac{\partial z}{\partial x} \right|_0 (x - 1) + \left. \frac{\partial z}{\partial y} \right|_0 (y - 1) = (x - 1) + 2(y - 1).$$

2. Give the linearization of $f(x, y) = e^x + x + y$ at $(0,0)$.

Answer: The tangent approximation formula at the point (x_0, y_0, z_0) is

$$f(x, y) - f(x_0, y_0) \approx f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0).$$

(We usually abbreviate this as $\Delta z \approx f_x|_0 \Delta x + f_y|_0 \Delta y$.)

Linearization is just the following form of the tangent approximation formula

$$f(x, y) \approx f(x_0, y_0) + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0).$$

In our case,

$$f_x(x, y) = e^x + 1 \text{ and } f_y(x, y) = 1 \Rightarrow f(0, 0) = 1, f_x(0, 0) = 2, f_y(0, 0) = 1$$

Thus, for $(x, y) \approx (0, 0)$ we have

$$f(x, y) \approx 1 + 2x + y.$$

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