

Uses of the Dot Product

1. Find the angle between the vectors $\mathbf{A} = \mathbf{i} + 8\mathbf{j}$ and $\mathbf{B} = \mathbf{i} + 2\mathbf{j}$.

Answer: As usual, call the angle in question θ . Since $\mathbf{A} \cdot \mathbf{B} = |\mathbf{A}||\mathbf{B}| \cos \theta$ we have

$$\cos \theta = \frac{\mathbf{A} \cdot \mathbf{B}}{|\mathbf{A}||\mathbf{B}|} = \frac{\langle 1, 8 \rangle \cdot \langle 1, 2 \rangle}{\sqrt{65} \sqrt{5}} = \frac{17}{5\sqrt{13}}$$

Thus, $\theta = \cos^{-1} \left(\frac{17}{5\sqrt{13}} \right)$.

2. Take points $P = (a, 1, -1)$, $Q = (0, 1, 1)$, $R = (a, -1, 3)$. For what value(s) of a is PQR a right angle?

Answer: We need $\overrightarrow{QP} \cdot \overrightarrow{QR} = 0 \Rightarrow \langle a, 0, -2 \rangle \cdot \langle a, -2, 2 \rangle = a^2 - 4 = 0 \Rightarrow a = \pm 2$.

3. Show that the diagonals of a parallelogram are perpendicular if and only if it is a rhombus, i.e., its four sides have equal lengths.

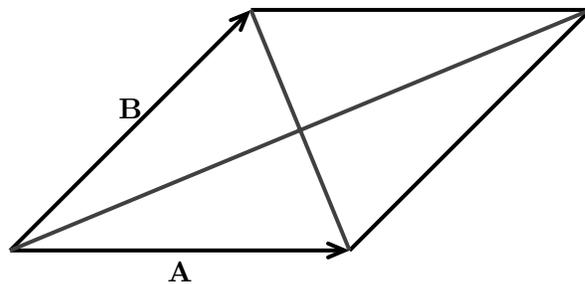
Answer: Let two adjacent sides of the parallelogram be the vectors \mathbf{A} and \mathbf{B} (as shown in the figure). Then we have the two diagonals are $\mathbf{A} + \mathbf{B}$ and $\mathbf{A} - \mathbf{B}$. We have

$$(\mathbf{A} + \mathbf{B}) \cdot (\mathbf{A} - \mathbf{B}) = \mathbf{A} \cdot \mathbf{A} - \mathbf{B} \cdot \mathbf{B}.$$

Therefore,

$$(\mathbf{A} + \mathbf{B}) \cdot (\mathbf{A} - \mathbf{B}) = 0 \Leftrightarrow \mathbf{A} \cdot \mathbf{A} = \mathbf{B} \cdot \mathbf{B}.$$

I.e., the diagonals are perpendicular if and only if two adjacent edges have equal lengths. In other words, if the parallelogram is a rhombus.



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18.02SC Multivariable Calculus
Fall 2010

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