

**JOEL LEWIS:** Hi. Welcome back to recitation. In lecture, among other things, you've been learning about computing components of one vector in the direction of another vector. So I have a straightforward problem about that for you here.

So we've got two vectors. The vector  $2\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ . And we've got the vector  $\mathbf{i} + \mathbf{j} + \mathbf{k}$ . And so what I'd like you to do for me is to compute the component of this first vector in the direction of the second vector. So why don't you pause the video, take some time to work that out, come back, and we can work on it together.

Welcome back. So hopefully you had some luck with this problem. Now this problem is pretty straightforward. Really all you have to do is remember what the definition of component is. And after that, it's smooth sailing.

So in particular, the component of one vector in the direction of another is the length of what you get when you project this vector onto that one, well, plus a sign, right? So if the projection is in the same direction, then it's positive, or if it's in the opposite direction, it's negative.

So let me draw a picture of what I mean by that. So if you have a vector  $\mathbf{v}$  and you have another vector  $\mathbf{w}$ , then the projection of  $\mathbf{v}$  onto  $\mathbf{w}$  is what you get if you make a perpendicular line there, and then it's just this vector here. So that's the projection. And then the component is the length of that projection. Again, with the sign if necessary.

And so we can see since this is going to be a right triangle here, we can see that this vector has length that's just given by the length of  $\mathbf{v}$ -- so the length-- rather, the component of  $\mathbf{v}$  in direction  $\mathbf{w}$  is-- so it's just the length of  $\mathbf{v}$ , right-- that's the length of the hypotenuse-- times the cosine of the angle between them, so it's times cosine of  $\theta$ .

Now this length of  $\mathbf{v}$  times cosine  $\theta$ , this should remind you of something. This looks very much like this formula we had for the dot product, right? So another way of writing this is to say that this is equal to  $\mathbf{v} \cdot \mathbf{w}$ --

so  $\mathbf{v} \cdot \mathbf{w}$  is the length of  $\mathbf{v}$  times the length of  $\mathbf{w}$  times the cosine of the angle. And so here we just have the length of  $\mathbf{v}$  times the cosine of the angle. So we have to divide this through by the length of  $\mathbf{w}$ . So this is what the component is, and this is the simple formula for it, if you're given  $\mathbf{v}$  and  $\mathbf{w}$  in some easy-to-use form. And indeed in this problem, we're given  $\mathbf{v}$  and  $\mathbf{w}$  just in their nice coordinate forms.

So we're given that our vector  $v$  that we want the component of is  $2i$  minus  $2j$  plus  $k$ . And the direction  $w$  that we're looking at is  $i$  plus  $j$  plus  $k$ . So in our case, we just have to compute these expressions  $v \cdot w$  and the length of  $w$  in order to put them into this formula, and then we'll be done.

So in our case,  $v \cdot w$ -- well, that's straightforward because we're given  $v$  and  $w$  in coordinates-- so this is just  $2$  times  $1$  plus minus  $2$  times  $1$  plus  $1$  times  $1$ . So that's, OK,  $2$  minus  $2$  plus  $1$  is  $1$ . And the length of  $w$ -- well, you know, it's just your usual length formula-- is  $1$  squared plus  $1$  squared plus  $1$  squared, the whole thing is square rooted, which is equal to the square root of  $3$ .

So we've got  $v \cdot w$  and we have the length of  $w$ , and so then we just need to put them right into this formula. That the component of  $v$  in the direction of  $w$  is given by this expression. So the component of-- I'm not going to write it out with  $i$ 's,  $j$ 's, and  $k$ 's, I'm going to write  $2, \text{ minus } 2, 1$ -- in direction  $[1, 1, 1]$  is, well we just have to divide the dot product by the length of the direction vector. So that's  $1$  divided by the square root of  $3$ .

So that's that, I'll end there.