

## Second derivative test

1. Find and classify all the critical points of

$$w = (x^3 + 1)(y^3 + 1).$$

**Answer:** Taking the first partials and setting them to 0:

$$w_x = 3x^2(y^3 + 1) = 0 \quad \text{and} \quad w_y = 3y^2(x^3 + 1) = 0.$$

The first equation implies  $x = 0$  or  $y = -1$ . We use these one at a time in the second equation.

If  $x = 0$  then  $w_y = 0 \Rightarrow y = 0 \Rightarrow (0, 0)$  is a critical point.

If  $y = -1$  then  $w_y = 0 \Rightarrow x^3 + 1 = 0 \Rightarrow x = -1 \Rightarrow (-1, -1)$  is a critical point.

The critical points are  $(0, 0)$  and  $(-1, -1)$ .

Taking second partials:

$$w_{xx} = 6x(y^3 + 1), \quad w_{xy} = 9x^2y^2, \quad w_{yy} = 6y(x^3 + 1).$$

We analyze each critical point in turn.

At  $(-1, -1)$ :  $A = w_{xx}(-1, -1) = 0$ ,  $B = w_{xy}(-1, -1) = 9$ ,  $C = w_{yy}(-1, -1) = 0$ . Therefore  $AC - B^2 = -81 < 0$ , which implies the critical point is a saddle.

At  $(0, 0)$ :  $A = w_{xx}(0, 0) = 0$ ,  $B = w_{xy}(0, 0) = 0$ ,  $C = w_{yy}(0, 0) = 0$ . Therefore  $AC - B^2 = 0$ . The second derivative test fails.

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