

## Problems: Green's Theorem

Calculate  $\oint_C -x^2y dx + xy^2 dy$ , where  $C$  is the circle of radius 2 centered on the origin.

**Answer:** Green's theorem tells us that if  $\mathbf{F} = \langle M, N \rangle$  and  $C$  is a positively oriented simple closed curve, then

$$\oint_C M dx + N dy = \iint_R N_x - M_y dA.$$

We let  $M = -x^2y$  and  $N = xy^2$  to get:

$$\begin{aligned} \oint_C -x^2y dx + xy^2 dy &= \iint_R y^2 - (-x^2) dA \\ &= \iint_R x^2 + y^2 dA \\ &= \int_0^{2\pi} \int_0^2 r^2 r dr d\theta \\ &= \int_0^{2\pi} \frac{8}{3} d\theta \\ &= \frac{16\pi}{3}. \end{aligned}$$

This result is  $4/3$  times the area  $\iint_R 1 dA$  of the circle, and so is a plausible answer.

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