

18.02 Problem Set 4

At MIT problem sets are referred to as 'psets'. You will see this term used occasionally within the problems sets.

The 18.02 psets are split into two parts 'part I' and 'part II'. The part I are all taken from the supplementary problems. You will find a link to the supplementary problems and solutions on this website. The intention is that these help the student develop some fluency with concepts and techniques. Students have access to the solutions while they do the problems, so they can check their work or get a little help as they do the problems. After you finish the problems go back and redo the ones for which you needed help from the solutions.

The part II problems are more involved. At MIT the students do not have access to the solutions while they work on the problems. They are encouraged to work together, but they have to write their solutions independently.

Part I (15 points)

At MIT the underlined problems must be done and turned in for grading. The 'Others' are *some* suggested choices for more practice.

A listing like '§1B : 2, 5b, 10' means do the indicated problems from supplementary problems section 1B.

1 Functions of several variables. Level Curves. Partial derivatives. Tangent plane. Linear Approximation

§2A: 1c, 2be, 3b, 5a; Others: 1abd, 2acd, 3ac, 4, 5b

§2B: 1b, 6, 9; Others: 1a, 2, 3, 5

2 Max-min problems. Least Squares.

§2F: 1a, 2; Others: 1b, 5;

§2G: 1c, 4; Others: 1ab

3 Second-derivative test. Boundaries. Applications

§2H: 1c, 3, 4, 6; Others: 1abe, 7

Part II (29 points)

Problem 1 (5: 3,2)

In this problem we'll use the 'Mathlet' **Functions of Two Variables** (with link on course webpage)

Let $f(u)$ be a function of one variable and let $F_c(x, t) = f(x - ct)$.

a) Use the plotting feature of the mathlet to examine the graphs of F_c in the cases $c = -1, 1, 2$ for $f(u) = (\cos(u))^2$

Note: you will need to use y for t when using *Input* to type the functions into the applet, since that's the only other letter allowed.

Then for $c = 2$ make a sketch by hand of the graphs of the single-variable function $x \rightarrow F_2(x, t)$ for the values $t = -1, 0, 1$.

b) If the variable t represents time, and x the position on a one-dimensional string, and f measures the displacement of the string perpendicular to the x-axis, what phenomenon is being described by the function $F_2(x, t)$?

Problem 2 (5: 2,2,1)

a) Find the curve of intersection of the surfaces $z = x^2 - y^2$ and $z = 2 + (x - y)^2$ in parametric form.

b) Find the angle of intersection of these two surfaces at the point $(2, 1, 3)$.

(The angle of intersection of two surfaces is defined to be the angle made by their tangent planes.)

c) Check that the tangent vector to the curve of intersection found in part(a) at the point $(2, 1, 3)$ lies in (i.e. is parallel to) the tangent plane of each of the two surfaces.

Problem 3 (7: 1,1,1,2,2)

Let $f(x, y) = \frac{4}{1 + x^2 + y^2}$ and let S be the surface given by the graph of $f(x, y)$.

a) Make a sketch of the surface S in 3-space, and also a (separate) sketch of the contour plot of f .

b) Let C_2 denote the curve in the xy-plane given by $\mathbf{r}_2(t) = \langle t, \frac{3}{2} - t^2 \rangle$; and let C denote the curve on the surface S which has C_2 as its shadow in the xy-plane. Find the parametric equations $\mathbf{r} = \mathbf{r}(t)$ for C .

(c) Sketch C in on the picture of S of part(a), and also C_2 on the contour plot of part(a).

d) Let $z(t)$ denote the z-component of the parametric equations $\mathbf{r} = \mathbf{r}(t)$ of C found in part(b). Find the points where $z(t)$ has its local maxima and minima, and add these in to the sketch of part(a).

e) Set up the function $h(t)$ which gives the square of the distance from the origin to a variable point on the curve C_2 , and then find the local maxima and minima of $h(t)$. How do these points relate to the maxima and minima found in part(d), and why?

Problem 4 (5)

Suppose that three non-negative numbers are restricted by the condition that the sum of their squares is equal to 27. Using critical point analysis, with 2nd derivative and/or boundary tests as needed, find the maximum and minimum values of the sum of their cubes.

Problem 5 (7: 4,3)

We return to problem 3 on p-set 1, where we computed the component of the wind vector \mathbf{w} which pointed *against* the wind after projecting \mathbf{w} twice.

The problem now is to find the combination of projections of $\mathbf{w} = \langle 1, 0 \rangle$ onto the vectors \mathbf{w}_1 and then onto \mathbf{w}_2 which yields the second projection which points *most strongly* into the wind, that is, the one with the most negative \mathbf{i} component.

In p-set 1, problem 3 we found the \mathbf{i} component of \mathbf{w}_2 was

$$\cos(\alpha) \cos(\beta) \cos(\alpha + \beta).$$

- What choice(s) of α and β will give the most negative \mathbf{i} component of \mathbf{w}_2 ?
- What fraction of the initial force of the wind is this?

This is the fraction of the force of the wind a sailboat can use to go against the wind with tacking.

Suggestion for Mathlet Experiments

Here are some suggested experiments to familiarize yourself with the *Level Curves* and *Partial Derivatives* selections in the applet **Functions of Two Variables**. These are optional, but people often find them useful for strengthening 3D visualization.

With the function $f(x, y) = y^2 - x^2$ start to familiarize yourself with the applet. Try out different windows, in addition to $[-2, 2] \times [-2, 2]$, to get a better picture of the part of the graph around the saddle point. Read the directions given, and then try out the different input and mouse-controlled features to see how they work. (Note also that the surface graph may be rotated in different directions to get a better view of the features of interest.) Also try these out on the given default example $f(x, y) = x(x - 1)(x - 2) + (y - 1)(x - y)$ for more practice. Then:

Using the *Partial Derivatives* applet for the function $f(x, y) = y^2 - x^2$ describe the behavior of f_x and f_y when you start at the saddle point (0,0) on the contour plot and then move the point in the following directions: E, NE, N, NW, W, SW, S, and SE (where e.g. SE means along the ray $\theta = -\frac{\pi}{4}$, etc.). Check the geometric pictures against the numerical results. If an intrepid mountaineer were moving along this surface, what would the partial derivative information be telling you about the rate of climb/descent on the mountain in the x- and y-directions for the E, N, W, S routes on the mountain?

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