

Partial derivatives

1. Let $f(x, y) = e^{(x^2+y^2)} + x^2 + y^2 + xy + 2y + 3$.

a) Compute $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.

b) Show the second partials can be computed in any order. That is,

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}.$$

c) Find $\frac{\partial f}{\partial x}(1, 3)$.

Answer: a) $\frac{\partial f}{\partial x} = 2xe^{(x^2+y^2)} + 2x + y$, $\frac{\partial f}{\partial y} = 2ye^{(x^2+y^2)} + 2y + x + 2$.

b) To compute $\frac{\partial^2 f}{\partial x \partial y}$ we compute the partial with respect to x of $\frac{\partial f}{\partial y}$.

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x} \left(\frac{\partial f}{\partial y} \right) = \frac{\partial}{\partial x} \left(2ye^{(x^2+y^2)} + 2y + x + 2 \right) = 4xye^{(x^2+y^2)} + 1.$$

Likewise

$$\frac{\partial^2 f}{\partial y \partial x} = \frac{\partial}{\partial y} \left(\frac{\partial f}{\partial x} \right) = \frac{\partial}{\partial y} \left(2xe^{(x^2+y^2)} + 2x + y \right) = 4xye^{(x^2+y^2)} + 1.$$

We have shown the order of differentiation didn't matter.

c) Evaluating $\frac{\partial f}{\partial x}(1, 3) = 2e^{10} + 5$.

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