

Problems: Elliptic Paraboloid

1. Compute the gradient of $w = x^2 + 5y^2$.

Answer: :

$$\nabla w = \left\langle \frac{\partial w}{\partial x}, \frac{\partial w}{\partial y} \right\rangle = \langle 2x, 10y \rangle.$$

2. Show that ∇w is perpendicular to the level curves of w at the points $(x_0, 0)$.

Answer: At $(x_0, 0)$, $\nabla w = \langle 2x_0, 0 \rangle$.

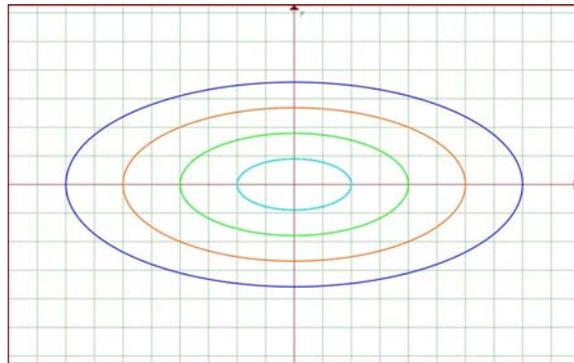


Figure 1: The level curves of $w = x^2 + 5y^2$.

In general, the level curves of w have equation $x^2 + 5y^2 = k$; each one is an ellipse whose major axis coincides with the x axis. Hence, the horizontal vector $\nabla w = \langle 2x_0, 0 \rangle$ will be normal to the level curve at the point $(x_0, 0)$.

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