

Problems: Flux Through Surfaces

Let $\mathbf{F} = \langle x, y, z \rangle$.

1. Find the flux of \mathbf{F} through the square with vertices $(0, 0, 0)$, $(1, 0, 0)$, $(1, 1, 0)$, $(0, 1, 0)$.

Answer: The square in question lies in the plane $z = 0$, so $\mathbf{n} = \langle 0, 0, 1 \rangle$. $\mathbf{F} \cdot \mathbf{n} = z = 0$ on the whole square, so the flux is zero.

2. Find the flux of \mathbf{F} through the square with vertices $(0, 0, 1)$, $(1, 0, 1)$, $(1, 1, 1)$, $(0, 1, 1)$.

Answer: Again $\mathbf{n} = \langle 0, 0, 1 \rangle$ and $\mathbf{F} \cdot \mathbf{n} = z$.

$$\text{Flux} = \iint_S \mathbf{F} \cdot \mathbf{n} dS = \int_0^1 \int_0^1 1 dx dy = 1.$$

3. Find the flux of \mathbf{F} through the surface $x^2 + y^2 = 1$ with $0 \leq z \leq 1$.

Answer: Here $\mathbf{n} = \langle x, y, 0 \rangle$, so $\mathbf{F} \cdot \mathbf{n} = x^2 + y^2 = 1$. We can parametrize the surface by $x = \cos \theta$, $y = \sin \theta$ with $dS = d\theta dz$ and integrate, or we can observe that the result of that calculation will just be the surface area of the cylinder. Flux = 2π .

MIT OpenCourseWare
<http://ocw.mit.edu>

18.02SC Multivariable Calculus
Fall 2010

For information about citing these materials or our Terms of Use, visit: <http://ocw.mit.edu/terms>.