

18.02SC Multivariable Calculus, Fall 2010
Transcript – Recitation 62, Del and the Product Rule

Welcome back to recitation. In this video what I'd like you to do is work on proving the following product rule for the del operator. So we're going to let capital F be a vector field and u be a scalar function. And we want to show the product rule for the del operator which, it's in quotes but it should remind you of the product rule we have for functions. And it is that del dot the quantity u times F-- so u is the scalar function and F is the vector field-- is actually equal to the gradient of u dotted with F plus u times del dot F. Where F again is the vector field. So why don't you take a moment to prove this fact and you can pause the video while doing that. And then when you're ready to check if your solution is correct, bring the video back up and I'll show you what I did.

OK, welcome back. Again what we wanted to do is prove this sort of pseudo product rule for the del operator. And what we're doing is we're trying to see what happens if you have a vector field and you multiply it by a scalar function and you apply the del operator to that. So we're going to see if we actually come up with what we should on the right hand side, which, since I'm calling it a rule, we really hope we do. In fact, we will.

So let me start off. What I'm going to do is I'm going to write symbolically what the left hand side means. And then we're going to break it up into pieces and show you get, in fact, what you would get on the right hand side. So symbolically what do we have? Oh actually, first I'm going to call F-- the components of F, as is usually done in lecture-- capitals P, Q, and R. So those will be the components of F. That's how we've been denoting this usually.

And notice that if we wanted F to be a vector field in two dimensions, we'd just make R 0, and then we'd have a vector field in two dimensions. So we can certainly do that if we want, but we're going to prove it in a little more-- in the three dimensional case. So remember that del dotted with any vector field is supposed to be symbolically-- what was written was you should think about this as del, del x, comma del, del y, comma del, del z, dotted with this vector field. $\nabla \cdot uF$.

Now what is this vector field? Because u is a scalar, when I multiply u by the vector field F, the components are going to be u capital P, comma u capital Q, comma u capital R. So those are my components. Now symbolically this is what we've seen when we're looking at the del operator acting on a vector field. So what do we actually do? Well, what we actually do, of course, is we take the x derivative of the first component, we take the y derivative of the second component, and we take the z

derivative of the third component. And then we add those together. So it's really a symbolic idea of a dot product. It's not a true dot product here.

But let's actually write down what we get there. We get $\frac{\partial}{\partial x}$ of the quantity uP , plus $\frac{\partial}{\partial y}$ of the quantity uQ , plus $\frac{\partial}{\partial z}$ of the quantity uR . That's exactly what this symbolically means is this, this what I've written in the next line down. So maybe I should write equals again. This is another equals. The top thing equals the next line down, equals the next line down. And just to have it nice and even I'll put the equals there, so when we look back it's easy to see.

Now how do I deal with this? Well, notice that u is a function and P is a function. Why is that? Because P was a component of a vector field. So it is just a function that is in the first component of the vector field. P , Q , and R are all individually functions that depend on x , y , and z . So here at this step I can actually take the regular product rule we have for functions. And I can do it in this one, this one, and this one.

And what I'm going to do, so I don't have to write $\frac{\partial}{\partial x}$ and $\frac{\partial}{\partial y}$ all over the place, I'm going to use the subscripts notation for derivatives. So I'm going to write what this actually equals is $u \text{ sub } x$ times P plus $u \text{ sub } x$. So here the $\frac{\partial}{\partial x}$ is now corresponding to a subscript. So notice that I've just used the product rule on functions at this step because u is a function and P is a function. I can do the same thing for the y derivative of uQ , I get $u \text{ sub } y Q$ plus $u \text{ sub } y$. And then the last component, I do the same thing for z . I get $u \text{ sub } z R$ plus $u \text{ sub } z$.

Now if I made a mistake it will become very apparent in the next moment. But I don't think I made a mistake. I want to remind us where we want to go and then we'll see if we have the pieces to get there. So I'm going to go back over to the side we have over here. And I want to remind you, we started off with a del operator acting on this vector field u times F . Capital F . And what we'd like to see is if we can get this to equal the gradient of u dotted with the vector field plus u times the del operator acting on F .

So let's see if we can first find components of the vector field and components of the gradient of u . And let's see where those are. They're actually very nicely placed here. Notice that this underlined component is the first component of gradient u and the first component of F . And this underlined component is the second component of gradient of u and the second component of F . And this third underlined component here is the third component of $\text{grad } u$ and the third component of F .

So if I take those three components together-- I'm going to write the equals up here-- I get exactly $\text{grad } u$ dotted with F . That's the first thing I get. And then I'm going to see what else I get. But let me just make sure you understand, look at these three underlined components together. You get u_x times P plus u_y times Q plus u_z times R . Well, u_x, u_y, u_z is the gradient vector field for u , and P, Q, R is F . So when I dot those, I get exactly $u_x P$ plus $u_y Q$ plus $u_z R$. You notice these two, or the dot product of this, gives you those three components.

And now there are three components remaining. Notice what they all have in common. They all have a u in the first spot. And then it's multiplied by P_x , and then here it's multiplied by Q_y , and here it's multiplied by R_z . But that is exactly-- P_x plus Q_y plus R_z is exactly the del operator acting on this vector field F . So this is something you've seen previously. So I'm going to do these as squiggles. That's exactly equal to u times the del operator of this vector field F .

So again let me remind you, $\text{del} \cdot F$ actually is going to give you P_x plus Q_y plus R_z . And then if I multiply that by a u , I get this u in front. So what I have done is for an arbitrary function u -- I guess I've assumed that function had first derivatives so that I could do all this stuff, and for a vector field that had first derivatives-- I showed that if I take the del operator of u times the vector field, I actually get the gradient of u dotted with the vector field plus u times the del operator of the vector field.

So that is what I wanted to show. If you remember, what I wanted to show was exactly that sort of pseudo product rule for this del operator. So I think that that is where I'll stop. I'm going to step off to the side so you can see it all again for a moment. But that's it. So that's where I'll stop.

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