

Finding Area Using Line Integrals

Use a line integral (and Green's Theorem) to find the area of the unit circle.

Answer: Recall that Green's Theorem tells us $\oint_C M dx + N dy = \iint_R N_x - M_y dA$. To find the area of the unit circle we let $M = 0$ and $N = x$ to get $\iint_R 1 dA = \oint_C x dy$.

We parametrize the circle by $x = \cos \theta$, $y = \sin \theta$, $0 < \theta \leq 2\pi$, so $x dy = \cos^2 \theta d\theta$. Then

$$\begin{aligned} \text{Area} &= \iint_R 1 dA \\ &= \oint_C x dy \\ &= \int_0^{2\pi} \cos^2 \theta d\theta \\ &= \int_0^{2\pi} \frac{1 + \cos 2\theta}{2} d\theta \\ &= \frac{1}{2} \left(\theta + \frac{1}{2} \sin 2\theta \right) \Big|_0^{2\pi} \\ &= \pi. \end{aligned}$$

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