

Least Squares Interpolation

1. Use the method of least squares to fit a line to the four data points

$$(0, 0), \quad (1, 2), \quad (2, 1), \quad (3, 4).$$

Answer: We are looking for the line $y = ax + b$ that best models the data. The deviation of a data point (x_i, y_i) from the model is

$$y_i - (ax_i + b).$$

The sum of the squares of the deviation is

$$\begin{aligned} D &= (0 - (a \cdot 0 + b))^2 + (2 - (a \cdot 1 + b))^2 + (1 - (a \cdot 2 + b))^2 + (4 - (a \cdot 3 + b))^2 \\ &= b^2 + (2 - a - b)^2 + (1 - 2a - b)^2 + (4 - 3a - b)^2. \end{aligned}$$

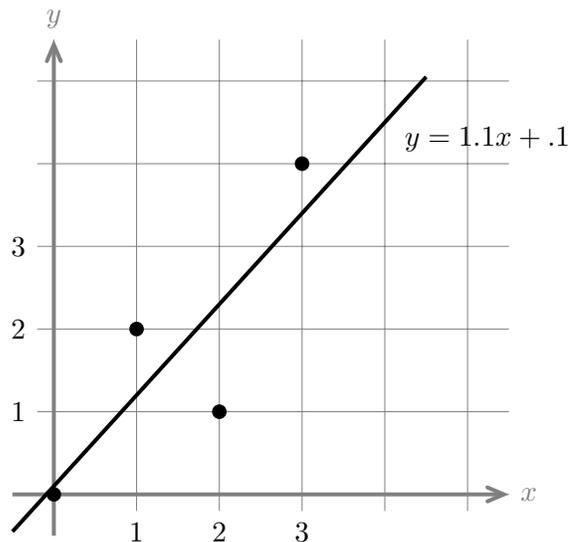
Taking derivatives and setting them to 0 gives

$$\begin{aligned} \frac{\partial D}{\partial a} &= -2(2 - a - b) - 4(1 - 2a - b) - 6(4 - 3a - b) = 0 \Rightarrow 28a + 12b = 32 \Rightarrow 14a + 6b = 16 \\ \frac{\partial D}{\partial b} &= 2b - 2(2 - a - b) - 2(1 - 2a - b) - 2(4 - 3a - b) = 0 \Rightarrow 12a + 8b = 14 \Rightarrow 6a + 4b = 7. \end{aligned}$$

This linear system of two equations in two unknowns is easy to solve. We get

$$a = \frac{11}{10}, \quad b = \frac{1}{10}.$$

Here is a plot of the problem.



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