

## 18.02 Problem Set 8

At MIT problem sets are referred to as 'psets'. You will see this term used occasionally within the problems sets.

The 18.02 psets are split into two parts 'part I' and 'part II'. The part I are all taken from the supplementary problems. You will find a link to the supplementary problems and solutions on this website. The intention is that these help the student develop some fluency with concepts and techniques. Students have access to the solutions while they do the problems, so they can check their work or get a little help as they do the problems. After you finish the problems go back and redo the ones for which you needed help from the solutions.

The part II problems are more involved. At MIT the students do not have access to the solutions while they work on the problems. They are encouraged to work together, but they have to write their solutions independently.

### Part I (12 points)

At MIT the underlined problems must be done and turned in for grading. The 'Others' are *some* suggested choices for more practice.

A listing like '§1B : 2, 5b, 10' means do the indicated problems from supplementary problems section 1B.

**1** Vector fields and Line Integrals in the plane.

§4A: 1d, 2bc, 3bd, 4; Others: 1b, 2a, 3ac

§4B: 1ab, 2b, 3; Others: 1cd, 2a

**2** Fundamental Theorem for line integrals, path independence

§4C: 1, 3; Others: 2

**3** Gradient fields, potential functions.

§4C: 5a(by V2 Method 1), 5b(by V2 Method 2), 6ab(by both methods); Others: 2

## Part II (23 points)

### Problem 1 (3 :1,2)

- a) Write down in  $xy$ -coordinates the vector field  $\mathbf{F}$  whose vector at  $P = (x, y)$  runs in the vertical direction from  $P$  to the line  $L : x + y = 1$ .
- b) Show *without calculation* that for this field,  $\int_C \mathbf{F} \cdot d\mathbf{r} \neq 0$ , if  $C$  is any positively oriented right triangle with legs parallel to the axes and hypotenuse on  $L$ .

### Problem 2 (5 :2,2,1)

Consider the vector field  $\mathbf{F} = -c \frac{x\mathbf{i} + y\mathbf{j}}{x^2 + y^2}$ . (This is the gravitational field of an infinitely long uniform wire along the  $z$ -axis.) Take  $c = 1$  and find by direct calculation (i.e., using a parametrization of  $C$ ) the work done by the gravitational field in moving a unit mass along each of the following paths  $C$  in the  $xy$ -plane.

- a)  $C$  is the half-line  $y = 1, x \geq 0$ .
- b)  $C$  is the circle of radius  $a$  with center at the origin, traced counterclockwise.
- c)  $C$  is the line from  $(0,1)$  to  $(1,0)$ .

### Problem 3 (4 :2,2)

- a) Let  $\mathbf{F} = -\nabla(\ln r)$ . Show that  $\mathbf{F}$  is the field in problem 2, when  $c = 1$ .
- b) Show that for any path  $C$  joining two points  $P_1$  and  $P_2$  the values of  $\int_C \mathbf{F} \cdot d\mathbf{r}$  depends only on the ratio  $r_2/r_1$ , where  $r_i$  is the distance of  $P_i$  to the origin.

### Problem 4 (6 :1,2,1,2)

Let  $\mathbf{F} = \nabla(x^2y + 2xy^2)$ . Let  $C$  be the quarter ellipse  $9x^2 + 4y^2 = 1$  running from the positive  $x$ -axis to the positive  $y$ -axis.

- a) Write the integral  $\int_C \mathbf{F} \cdot d\mathbf{r}$  in the form  $\int_C M dx + N dy$ .
- b) By parametrizing the curve  $C$  write the integral  $\int_C \mathbf{F} \cdot d\mathbf{r}$  as an explicit definite integral in  $t$ . (*Do not evaluate.*)
- c) Use the Fundamental Theorem to (easily) compute  $\int_C \mathbf{F} \cdot d\mathbf{r}$ .
- d) Use path independence to choose a different path and (again easily) compute  $\int_C \mathbf{F} \cdot d\mathbf{r}$ .

### Problem 5 (5 :1,2,2)

Let  $\mathbf{F} = xy\mathbf{i} + x^3\mathbf{j}$ .

- a) Show  $\mathbf{F}$  is not conservative.
- b) Try to find a potential function for  $\mathbf{F}$  using method 1 from the reading (section V2). What goes wrong?
- c) Answer the same question for method 2.

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